Process length scales as a framework for understanding flow, transport, and evolution of the karst Critical Zone

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Collaborators:

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“It is nice to know that the computer understands the problem, but I would like to understand it too.”

-Eugene Wigner

Two typical approaches to mathematical modeling in the geosciences

Descriptive:
Statistical models and black box models

Strength: Easy to apply, few parameters, may accurately predict system behavior.

Weakness: connection to physical processes is frequently unclear. Predictive models may fail outside of observed range.

Numerical simulations:
Models built from basic physics and chemistry

Strength: Direct connections to physical processes, assumptions clear.

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Simple models (toy models)

Metrics and analytical solutions developed from process-based analysis

How can we interpret the physical and chemical variations in flow at karst spring, cave stream, or drip site?

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Recession curve analysis

Reservoir models

System analysis

Neural networks

Numerical simulations:
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Reactive transport

Heat transport

Coupled conduit-matrix flow

CFD simulations

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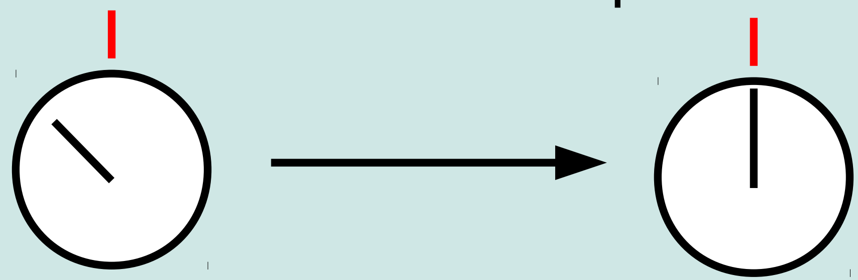
Characteristic time scales,
characteristic length scales, dimensionless metrics

Process Lengths:
A simple idea with many potential applications

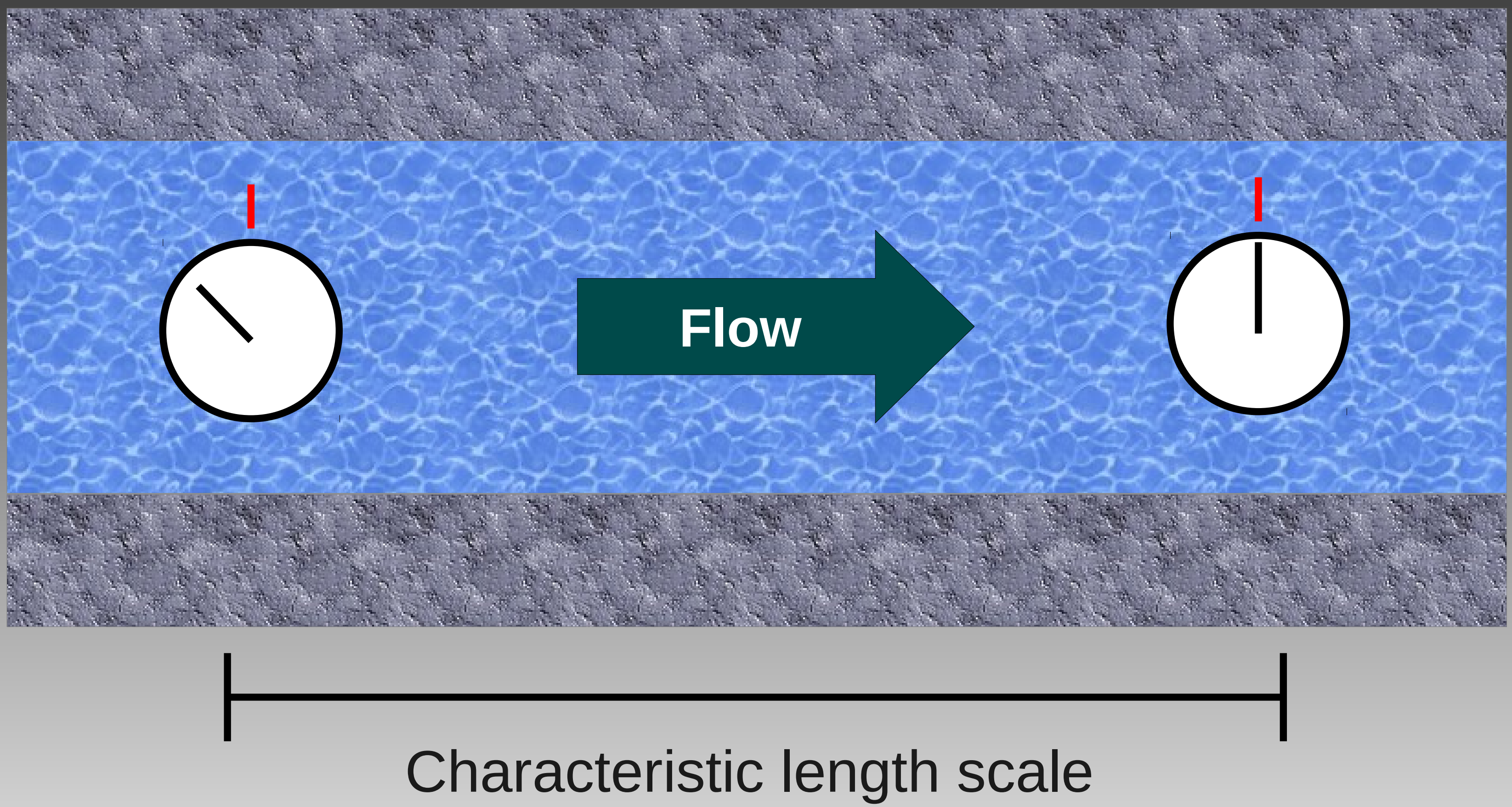
Two requirements:

1. A process that occurs over a particular time scale



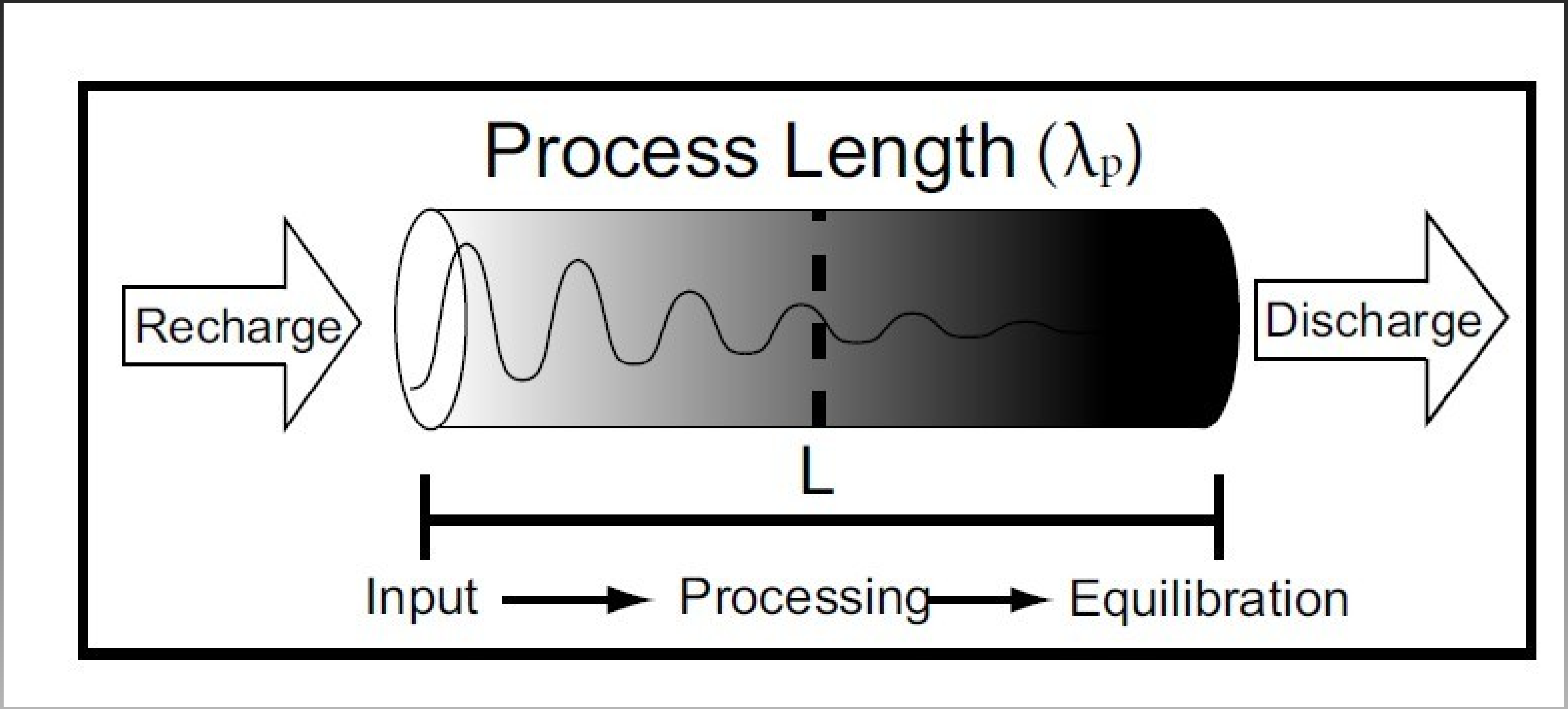
2. Flow that carries the process down a conduit (water or air)

A characteristic length scale emerges



Length = (Time Scale) X (Flow Velocity)

Does a given process produce variations at a flowpath outlet?



Covington et al., (2012). Process length scales and longitudinal damping in karst conduits. J. Geophys. Res.

Dissolutional Length Scales

$$\lambda_{\mathrm{dis}} = \frac{Q}{P\alpha} = \frac{V D_{\mathrm{H}}}{4\alpha},$$

Using laminar flow:

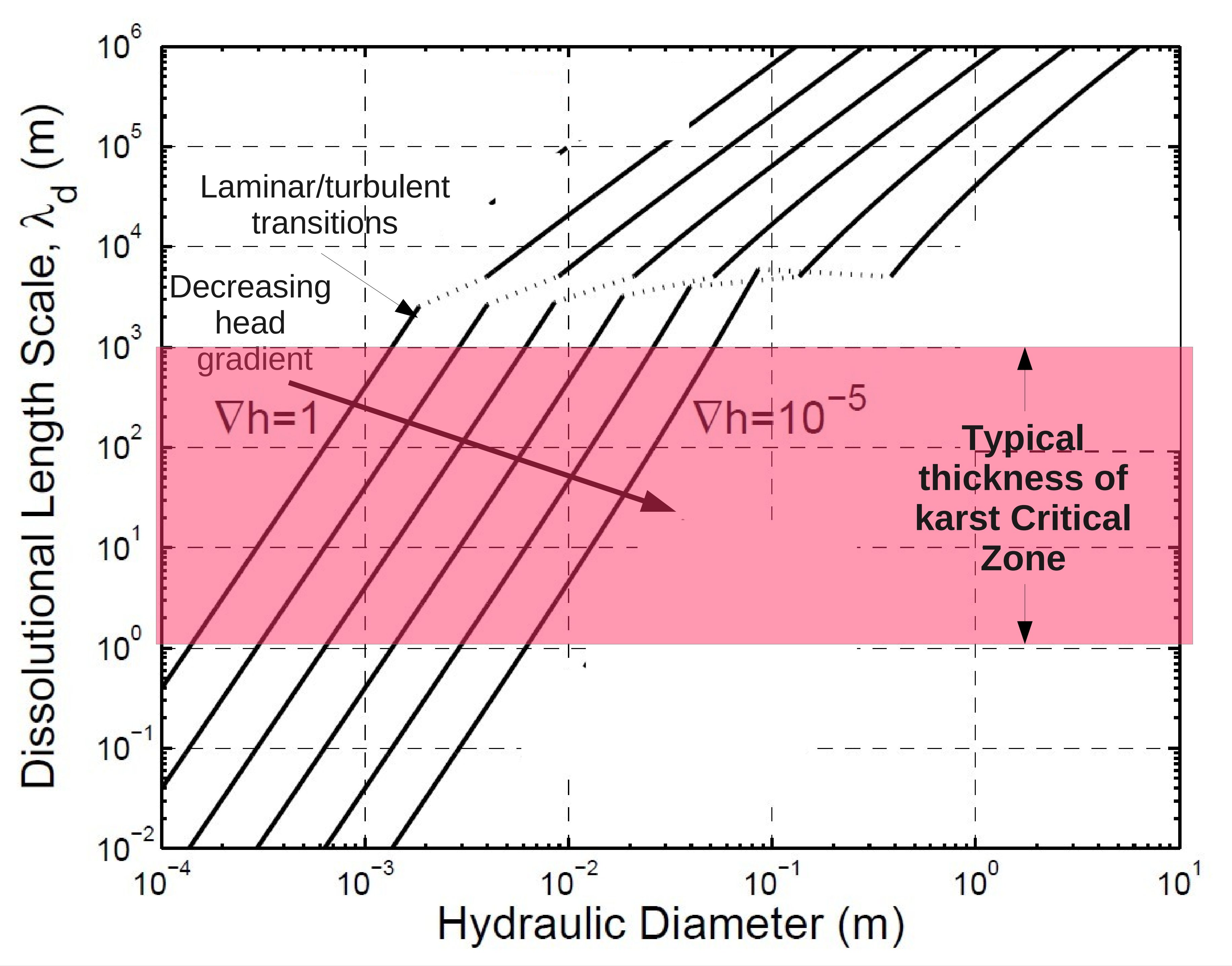
$$\lambda_{\mathrm{dis,lam}} = \frac{\rho g}{128\mu\alpha_{\mathrm{d}}}\nabla h D_{\mathrm{H}}^{3}.$$

Using turbulent flow:

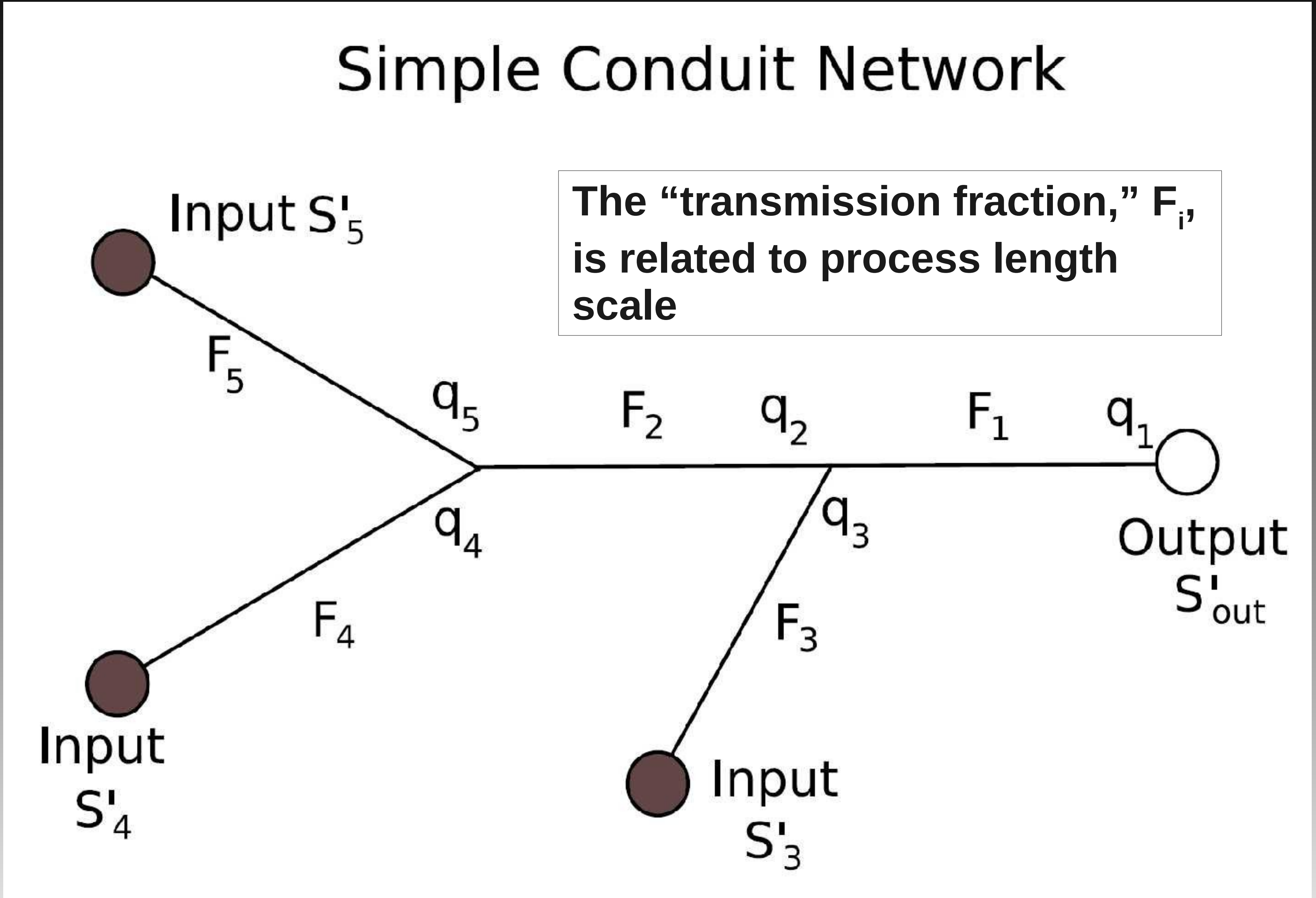
$$\lambda_{\mathrm{dis,turb}} = \frac{1}{4\alpha}\sqrt{\frac{2g\nabla h}{f}} D_{\mathrm{H}}^{3/2}.$$

Longitudinal profiles of concentration are exponential, with e-folding length, λ.

(Covington et al., 2012, J. Geophys. Res.)



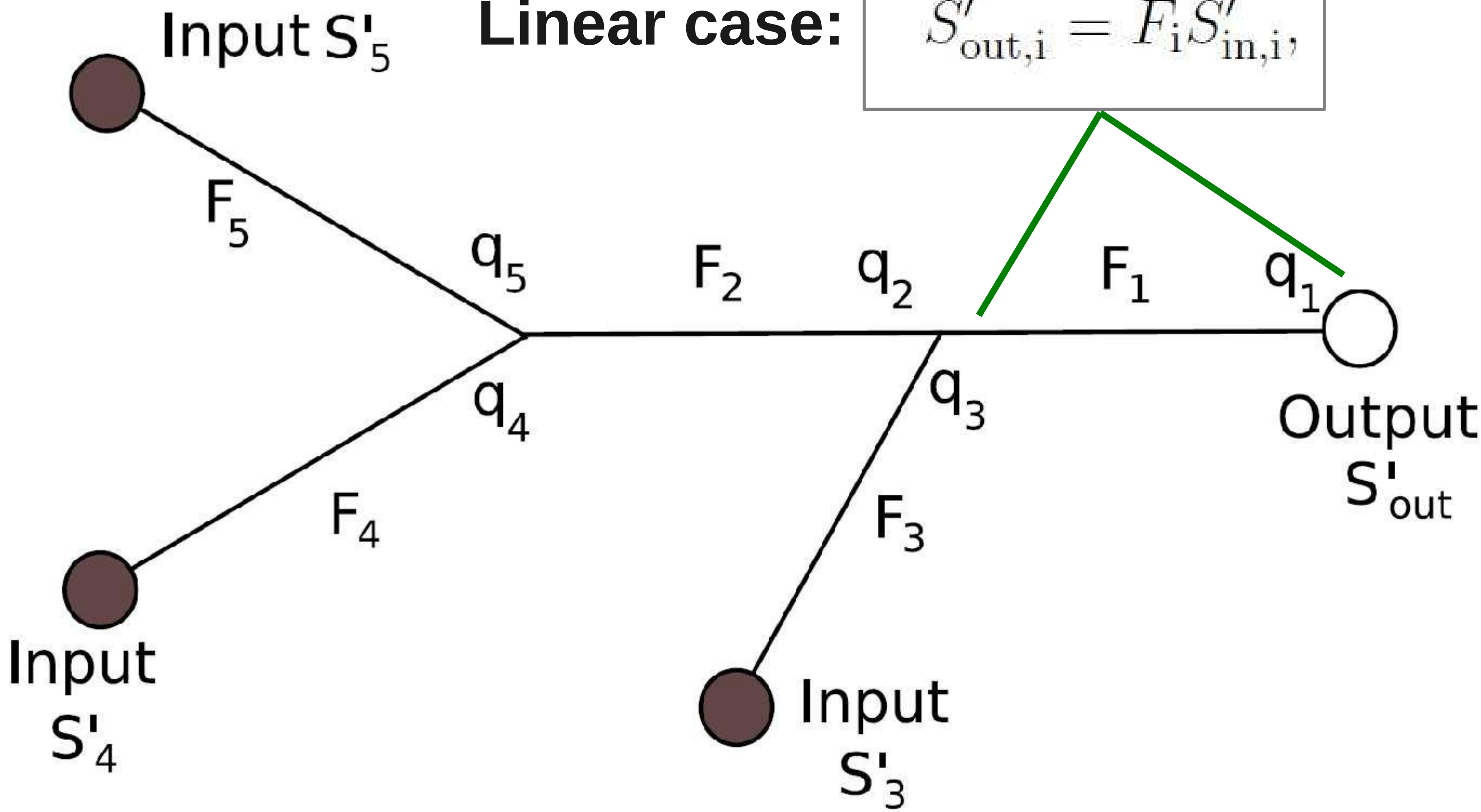
(Covington et al., 2012, J. Geophys. Res.)



(Covington et al., 2012, J. Geophys. Res.)

Simple Conduit Network

Linear case: $S'_{\mathrm{out,i}} = F_{\mathrm{i}} S'_{\mathrm{in,i}},$



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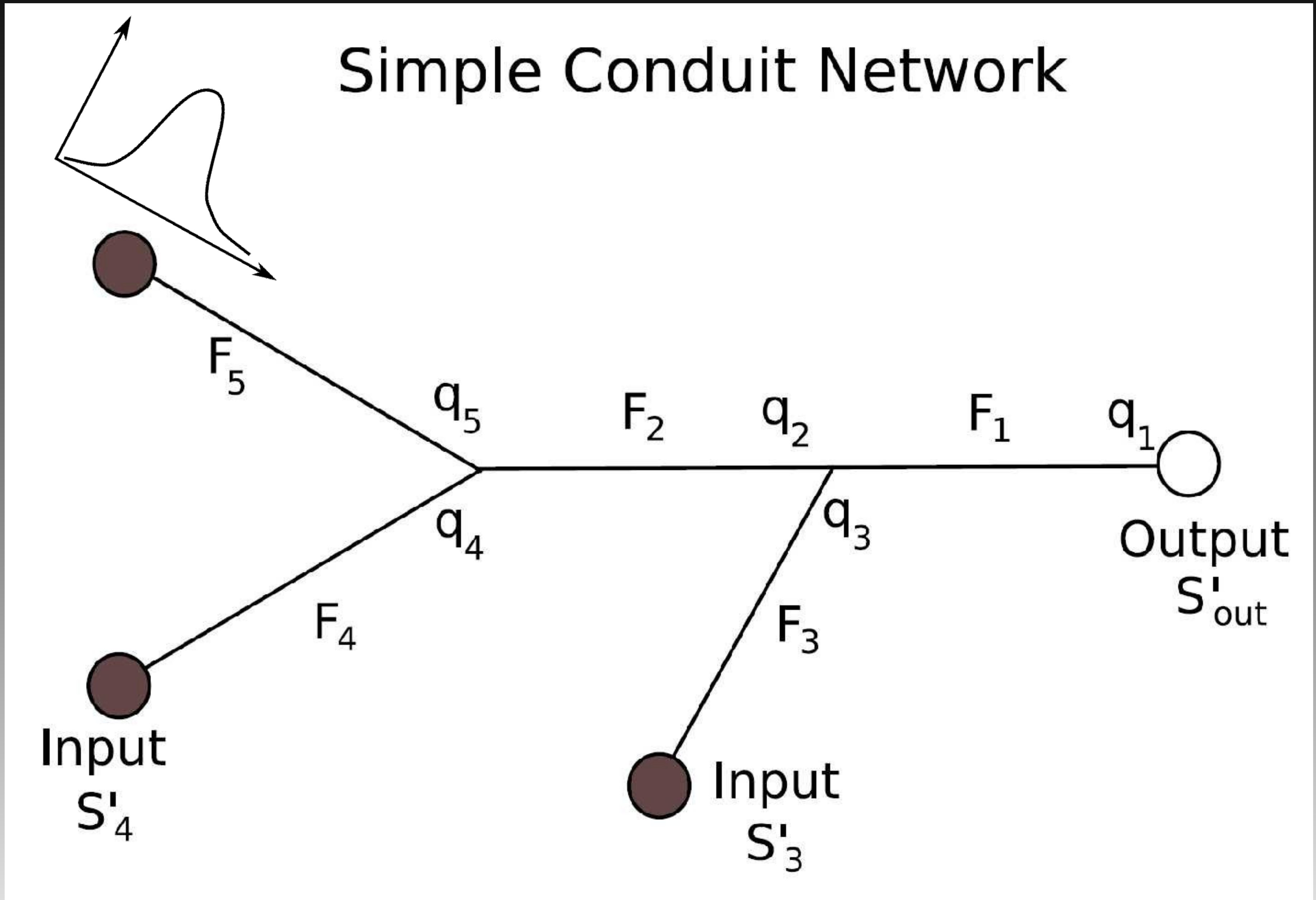

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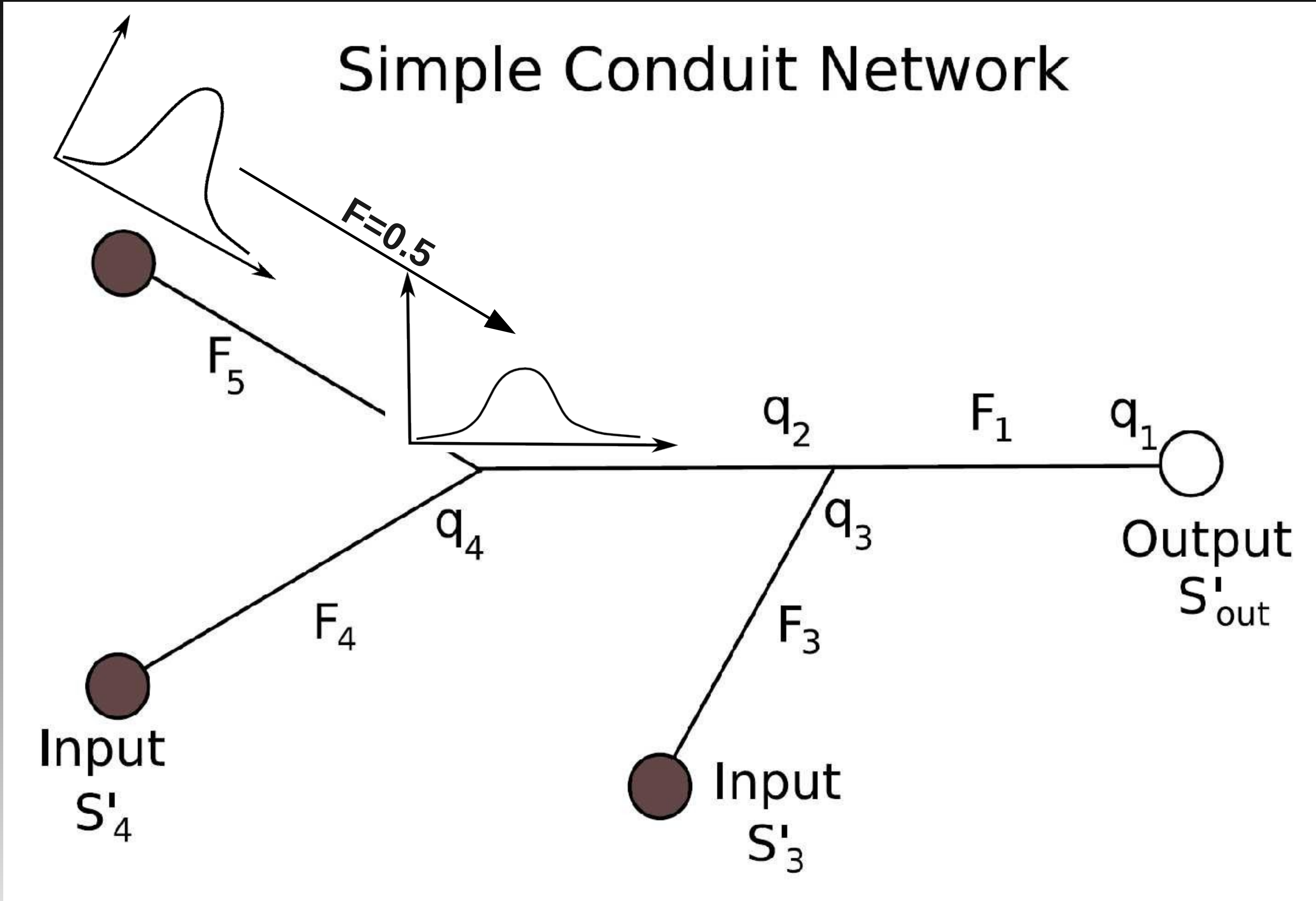

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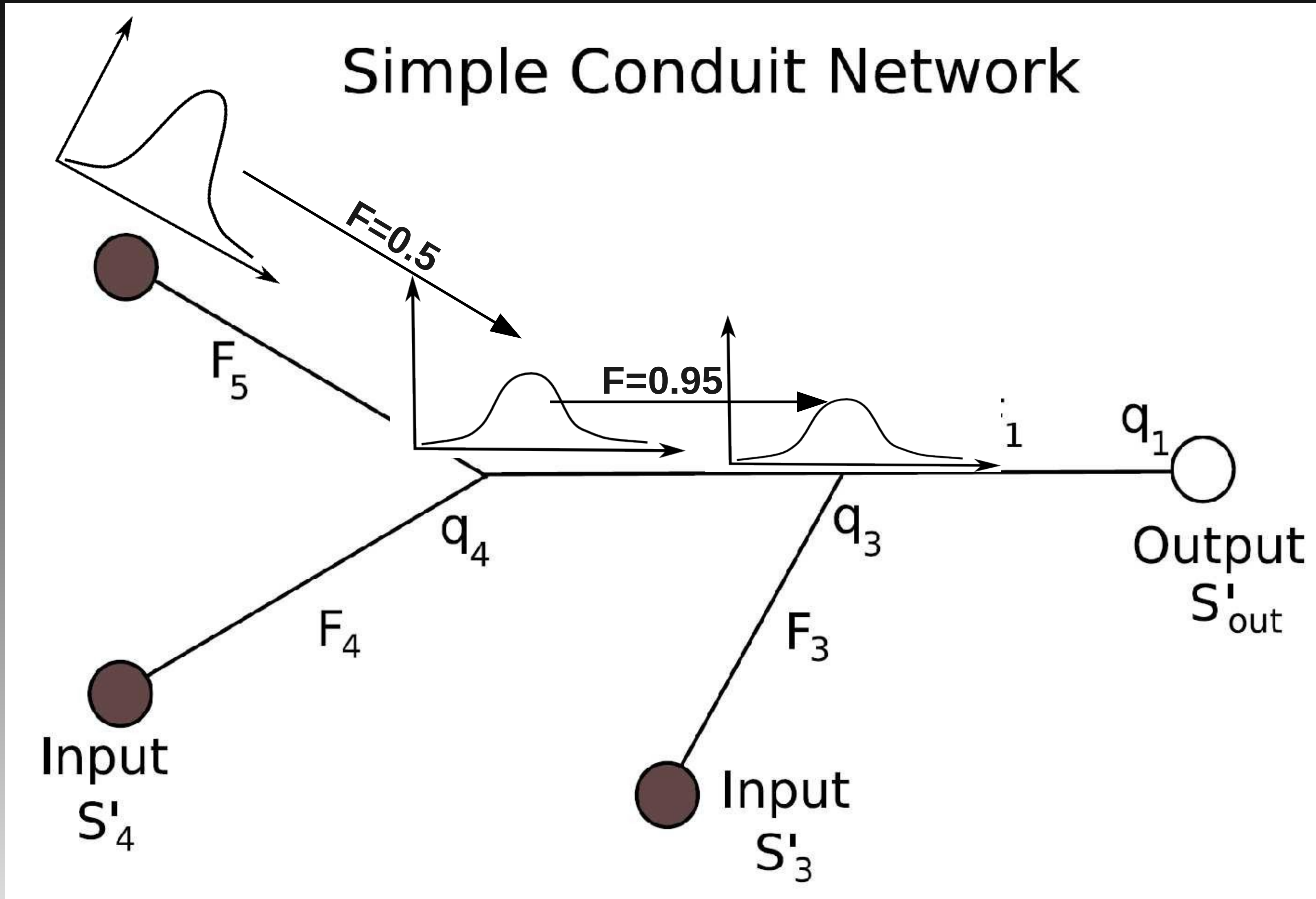

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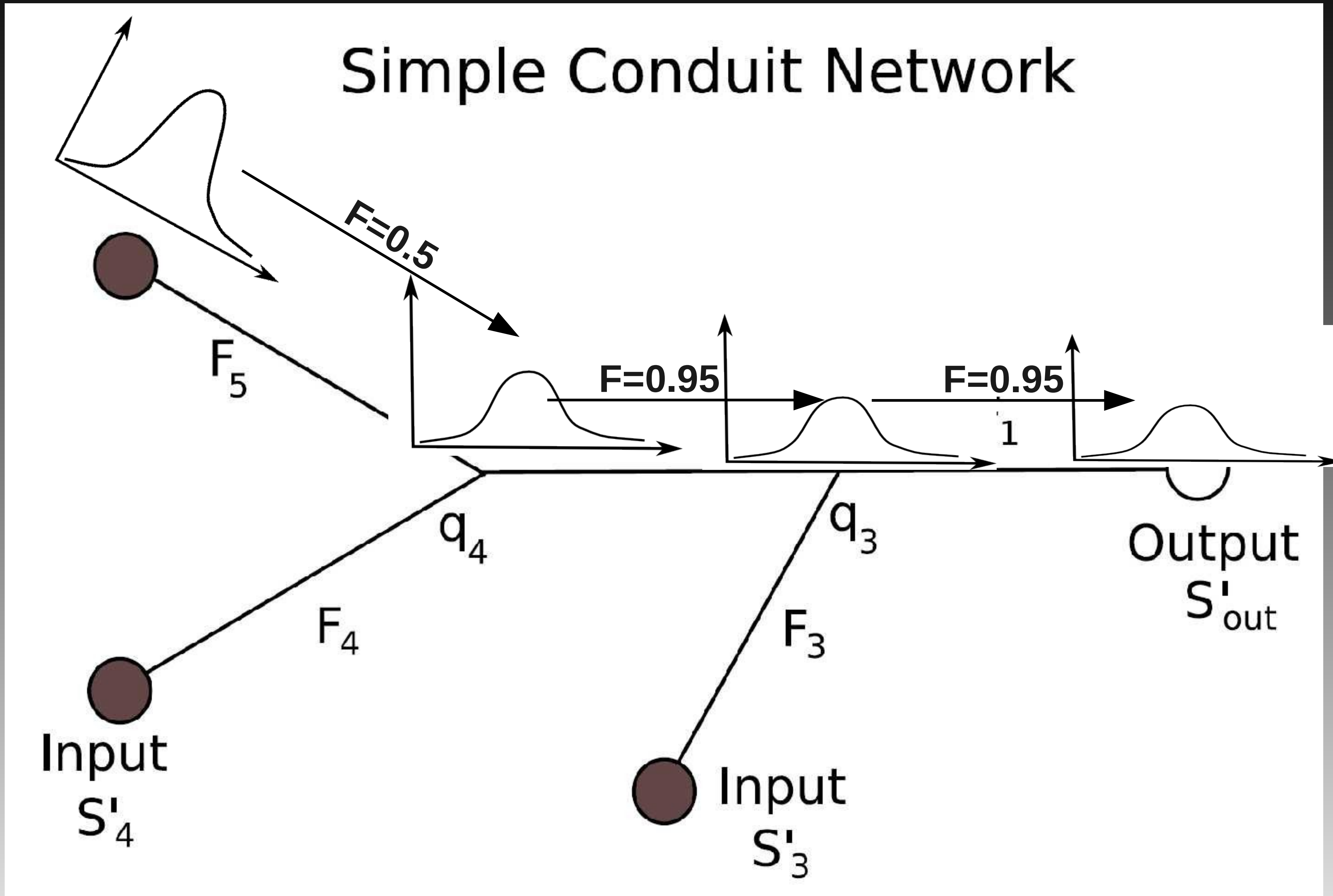

(Covington et al., 2012, J. Geophys. Res.)

Response of Conduit Networks

$$S'_{\text{out}} = S'_{\text{in}} \int_0^1 \phi_{\text{R}} F_{\text{path}} \; dF_{\text{path}}.$$

Physical Interpretation:
factors that control signal amplitude

1. Input signal amplitude
2. The capability of individual flow paths to transmit or dampen the signal
3. The distribution of flow among paths with different transmission factors

(Covington et al., 2012, J. Geophys. Res.)

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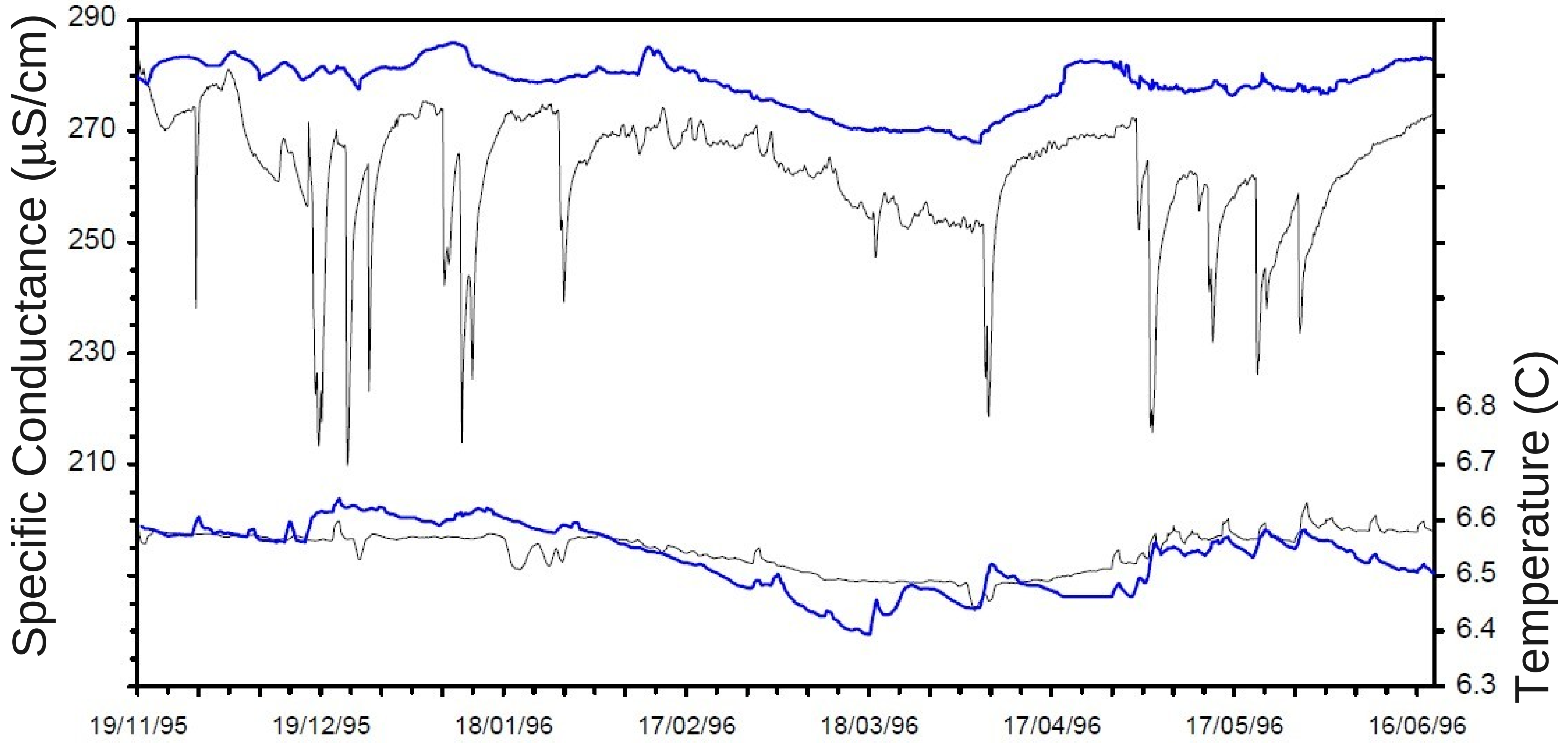
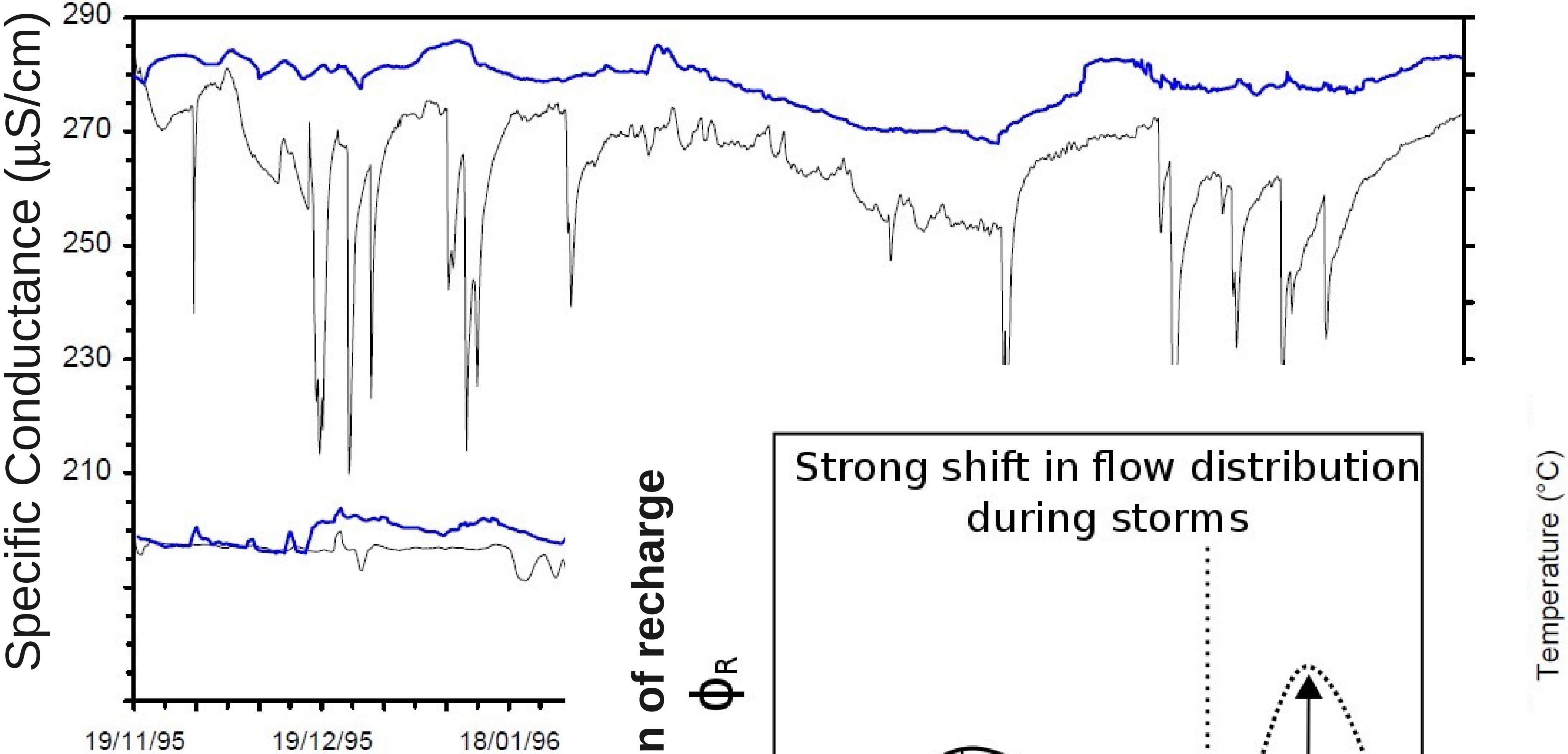
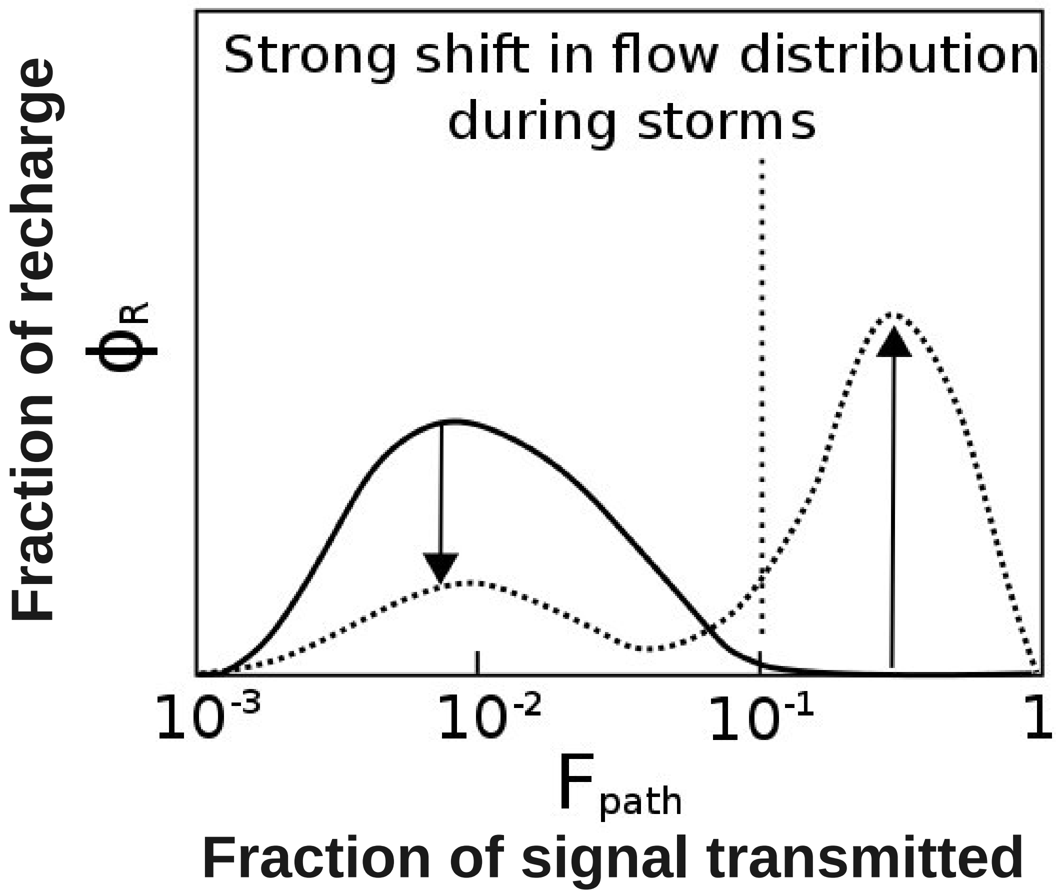


Borsato (1997). Data from two drip sites in Grotta di Ernesto





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Length scales and epikarst evolution

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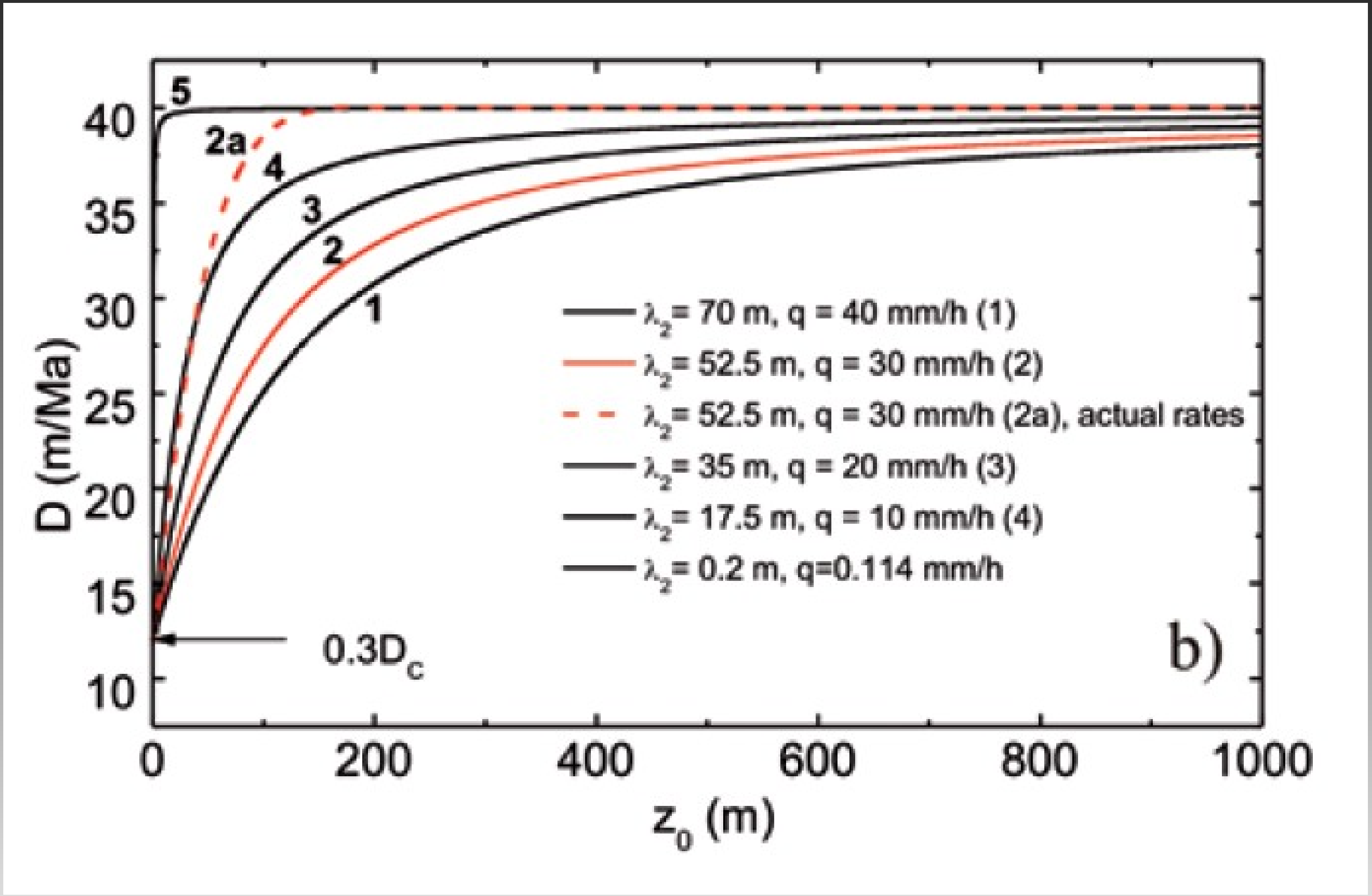
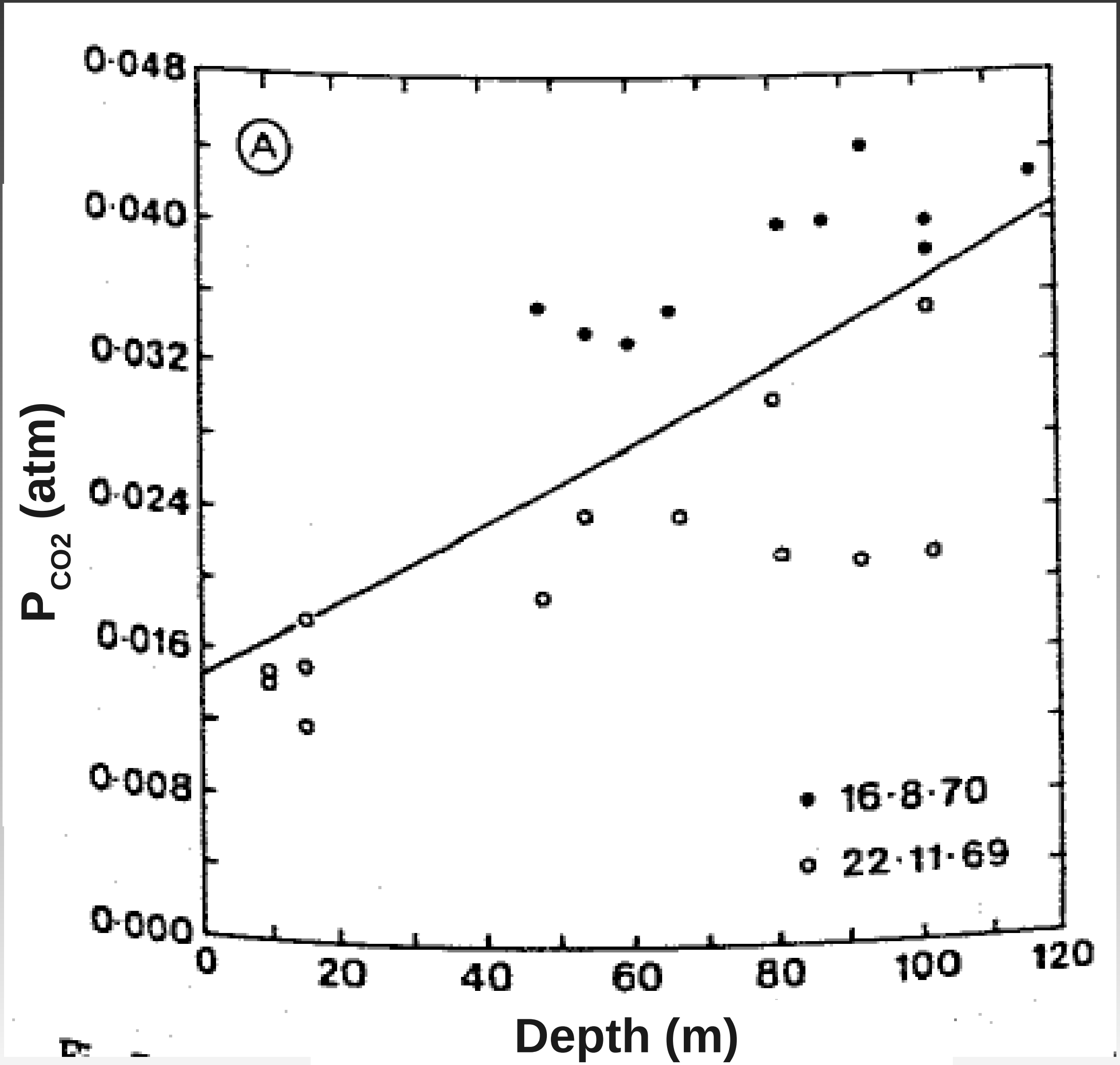


Gabrovšek (2007), Acta Carsologica

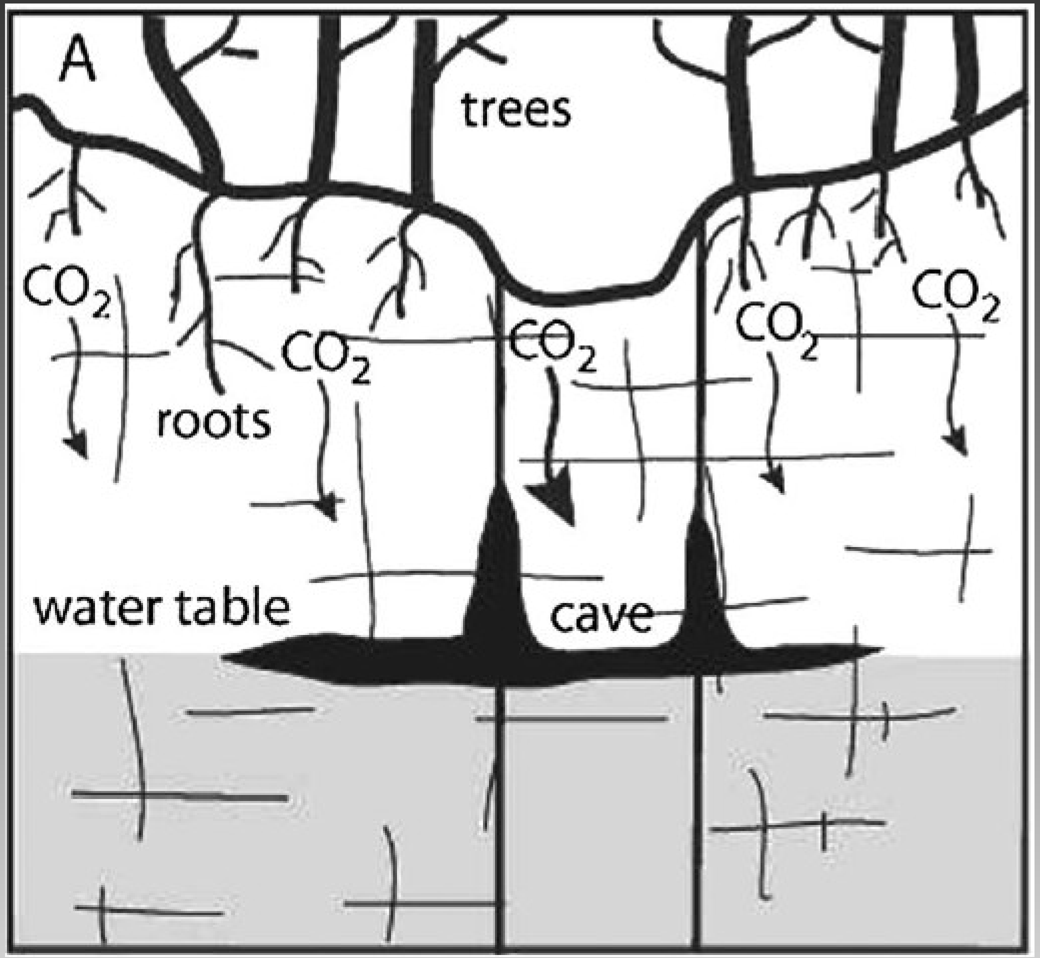
Length scales and epikarst evolution
What about CO_2?



Frequently, CO_2 increases with depth in the vadose zone

Atkinson (1977), Journal of hydrology.

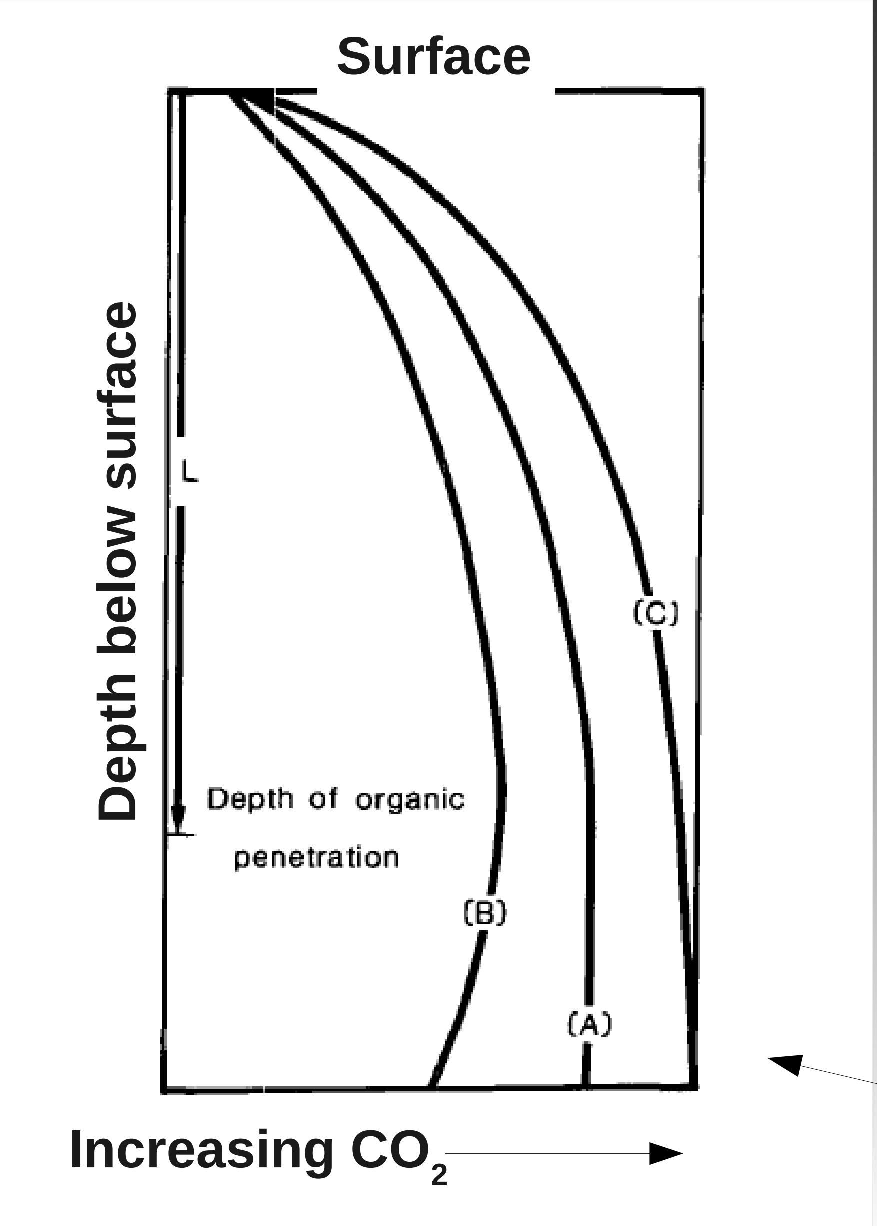
CO_2 at depth may be an important driver in speleogenesis and diagenesis



Gulley et al (2013). ESPL.

See also Whitaker and Smart (2007). Hydrol. Process.

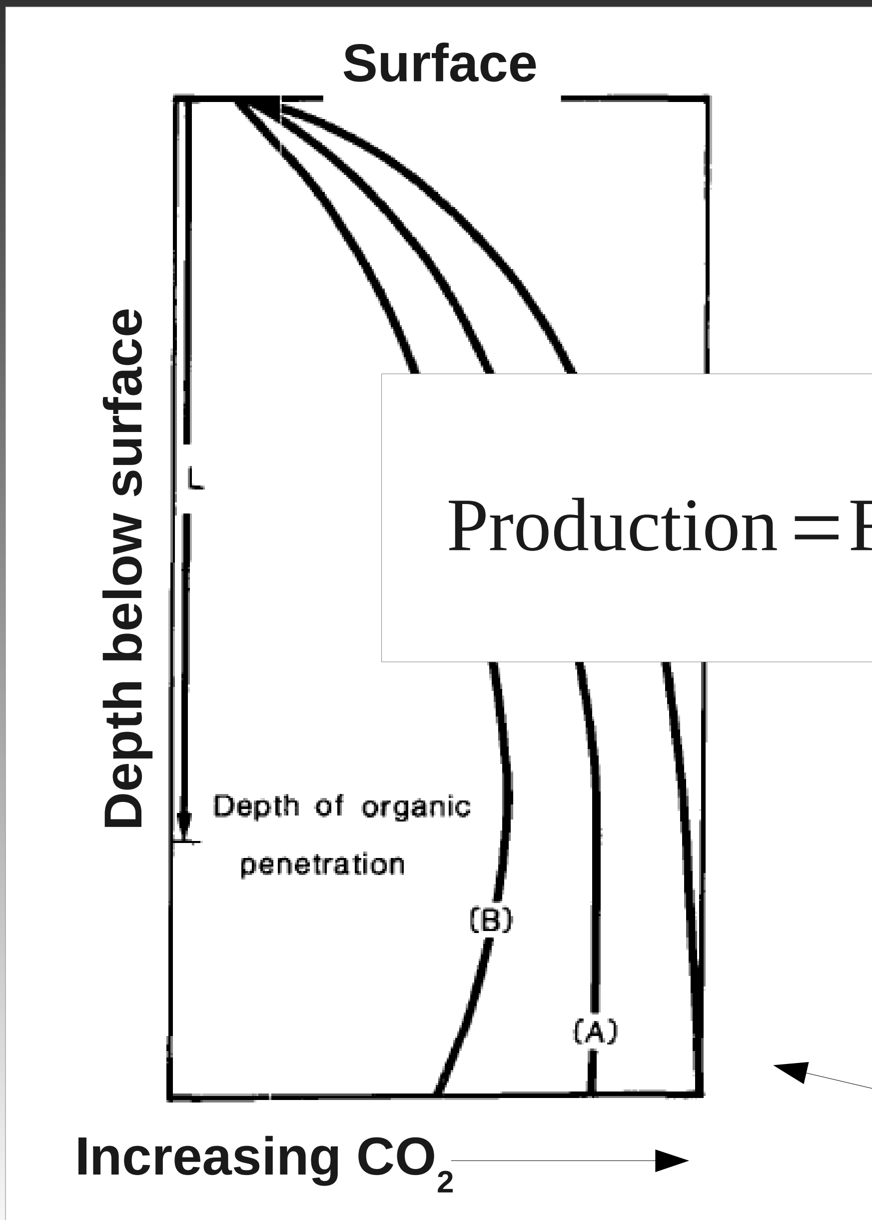
CO_2 production and diffusion: the basics



- Diffusion is a slow process
- With even a small amount of CO_2 production at depth, we can reach high concentrations.
- Without a significant CO_2 sink at depth, have to establish a significant gradient to allow diffusion to the surface.
- For the same CO_2 production, deeper vadose zones will have higher CO_2

Wood (1985). Geology.

CO_2 production and diffusion: the basics



$$\text{Production} = \text{Flux} \sim \frac{D\left(C_{deep} - C_{surface}\right)}{h}$$

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What about advection?

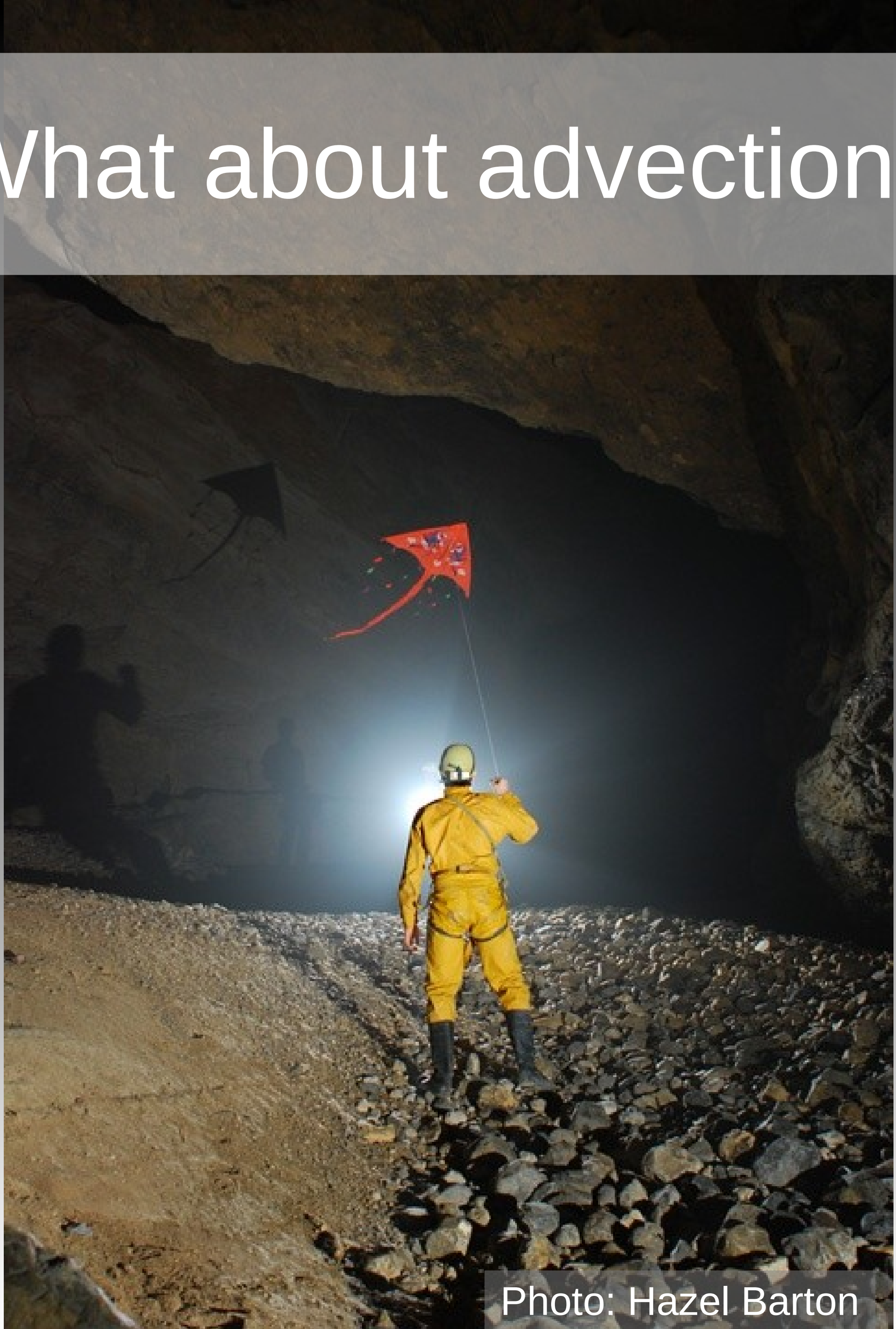

Photo: Hazel Barton

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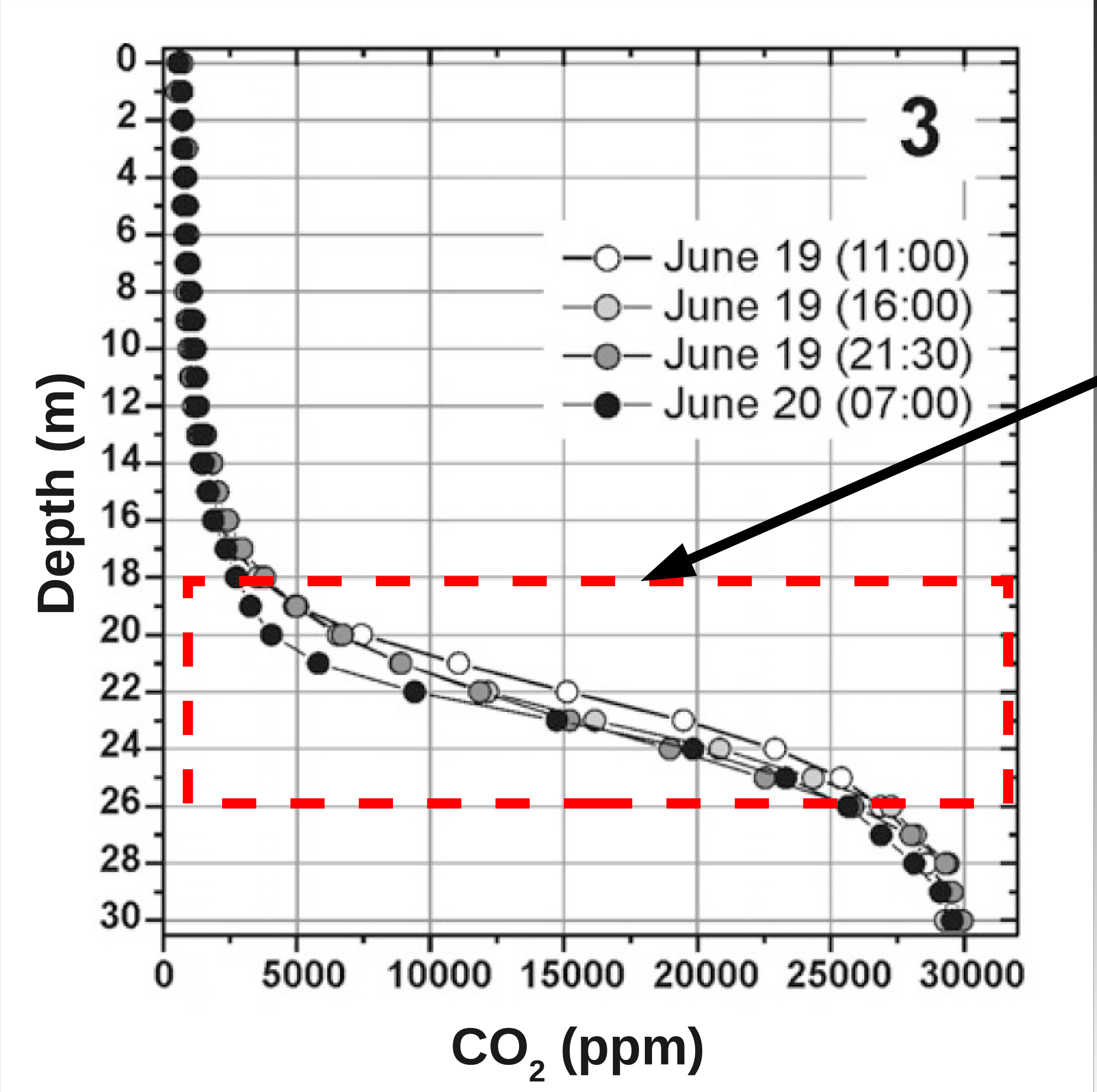
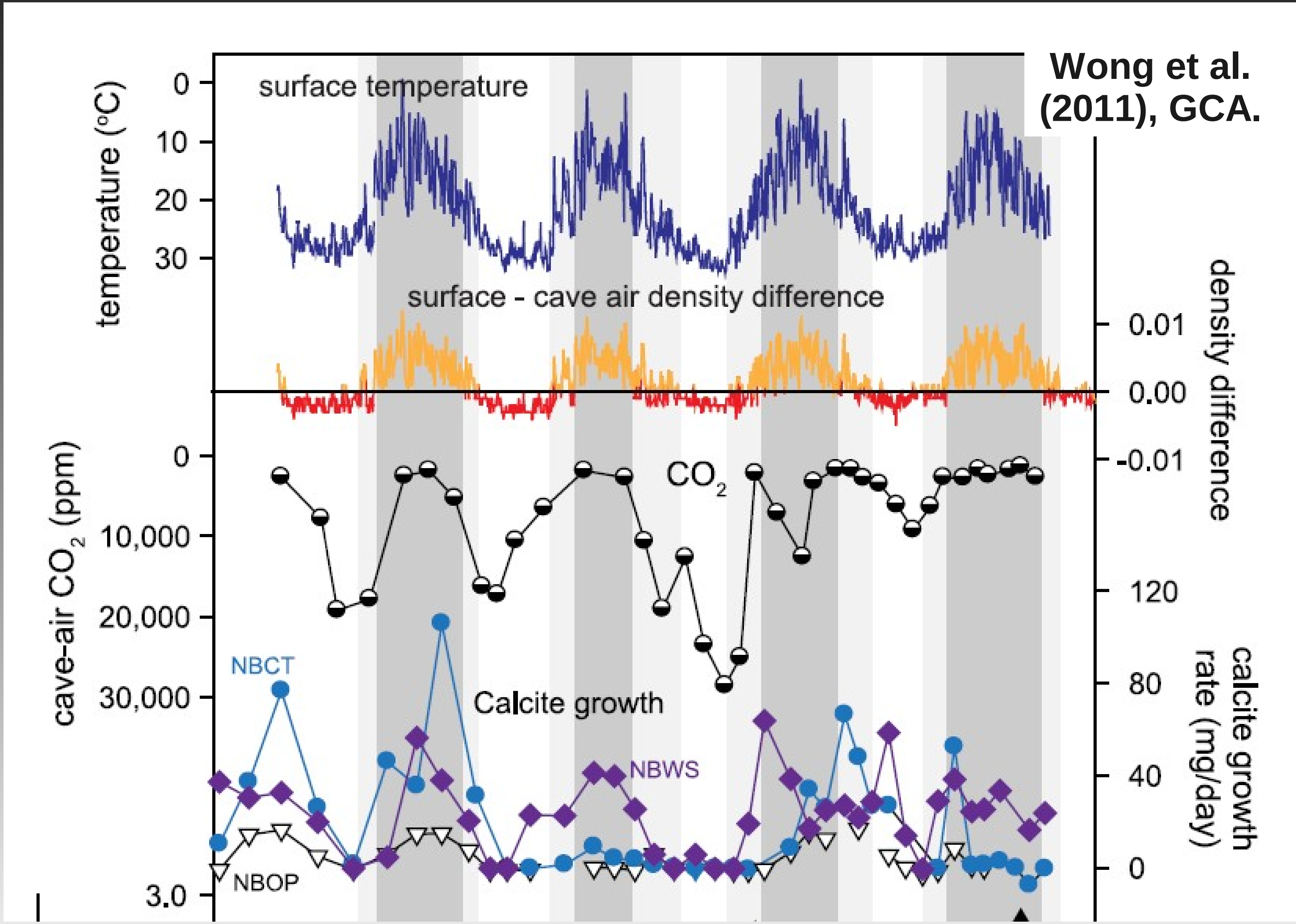


Diffusion abhors sharp edges!

Figure from Benavente et al. (2010). Vadose Zone J.

Seasonal airflow patterns as a strong control on cave CO_2



Wong et al. (2011), GCA.

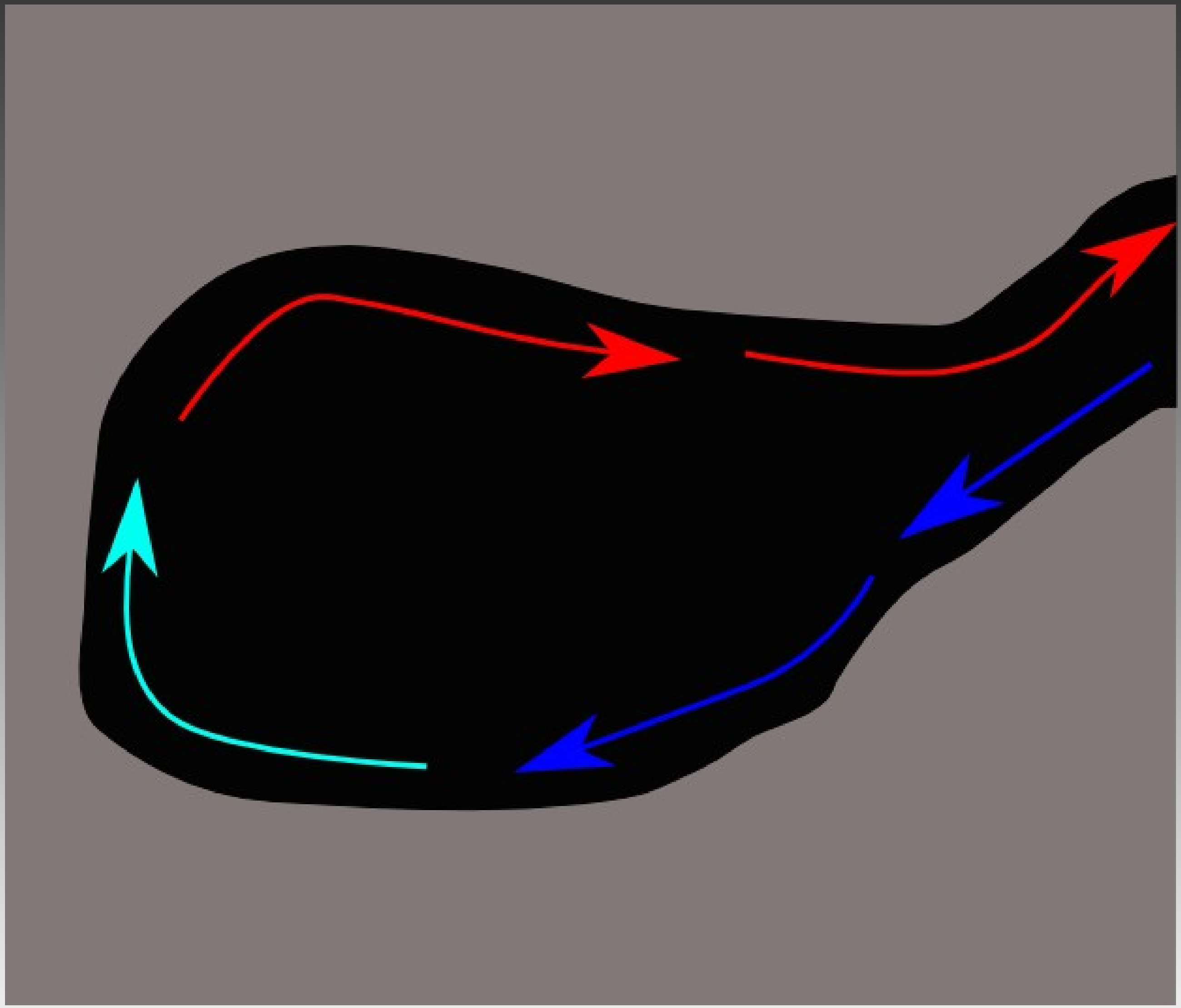
See also:

Frisia et al. (2011), GCA.

and

Breecker et al. (2012), GCA.

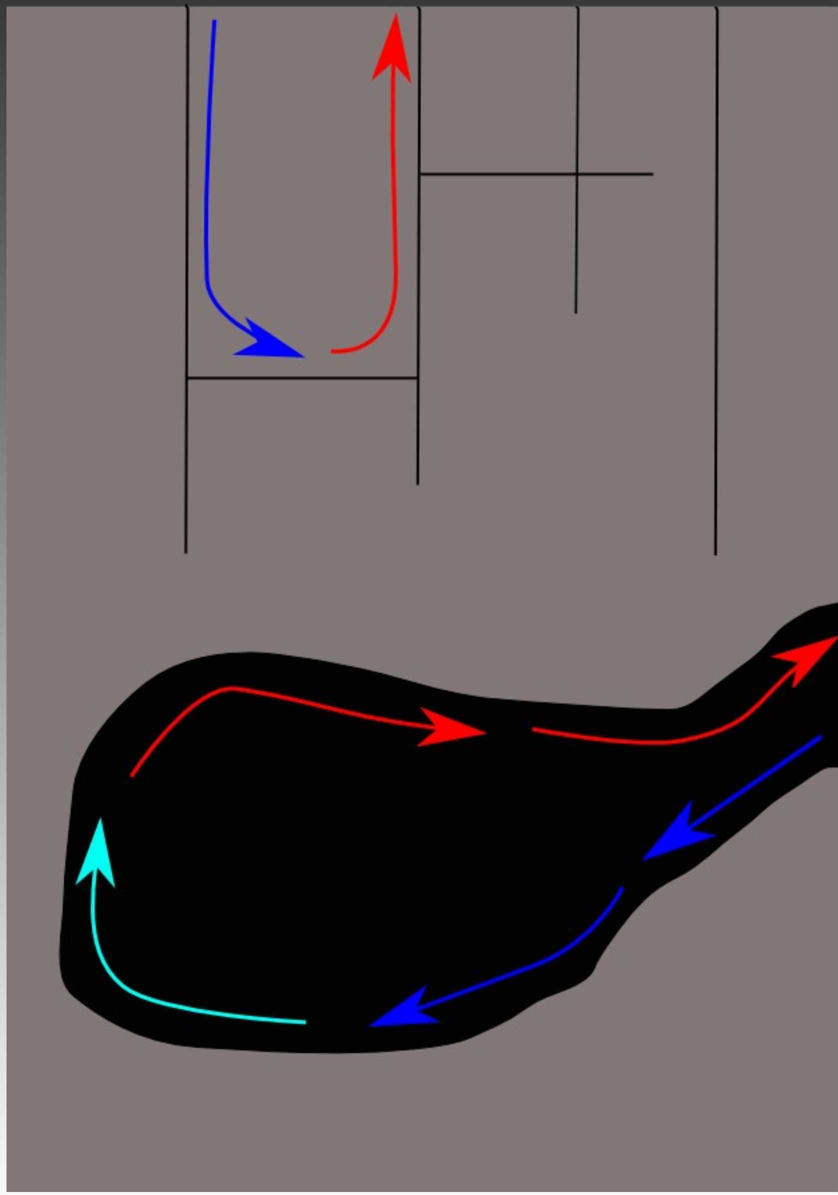
Modes of advective flow in karst
Entrance zone convection cells



Entrance with cool external temperatures

Modes of advective flow in karst

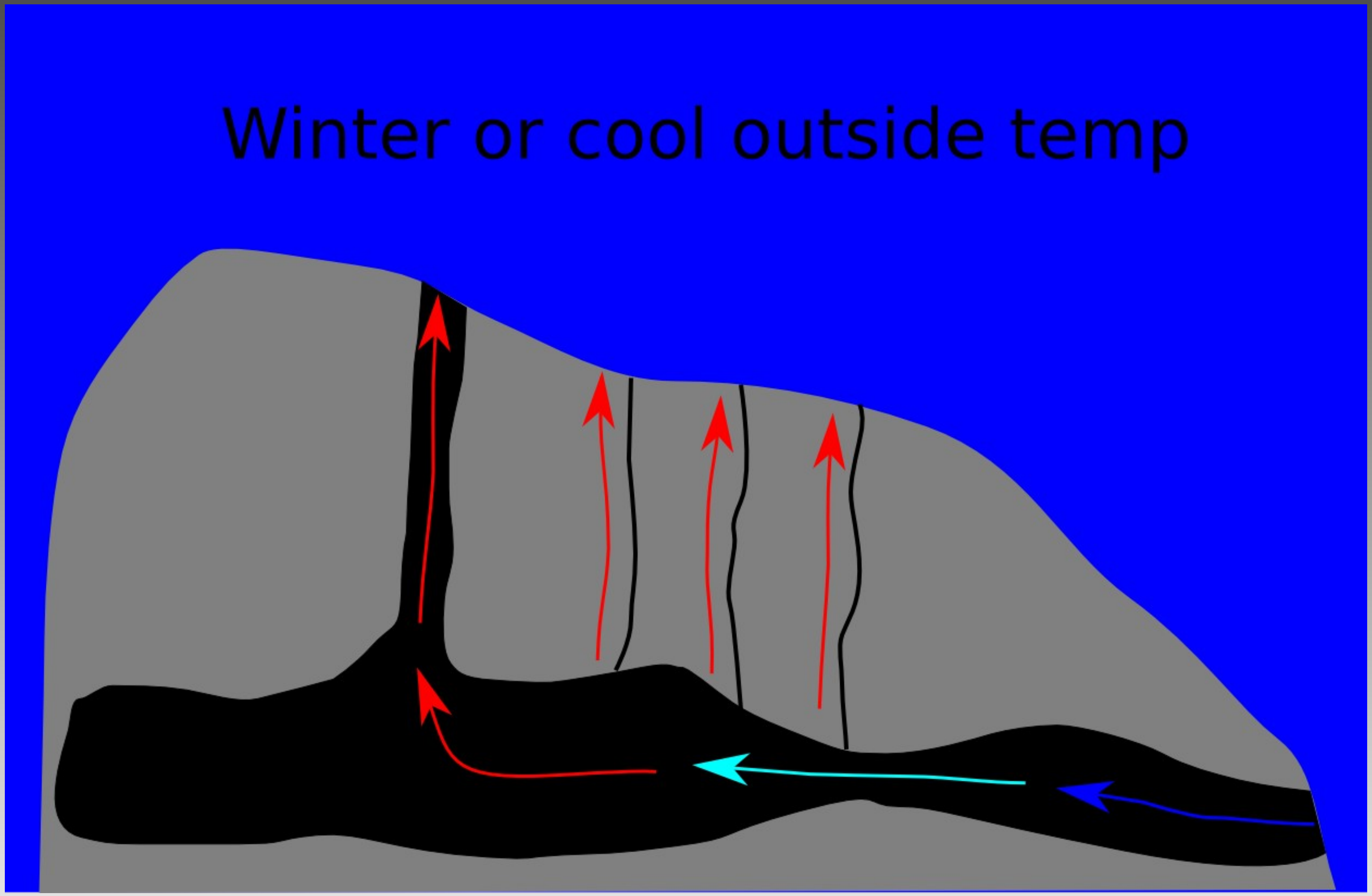
Entrance zone convection cells



Similar mechanisms may drive convective cells through fracture networks

Modes of advective flow in karst

Chimney-effect airflow



When outside temperature is **cooler** than cave temperature airflow is driven from ***lower*** to ***higher*** entrances

Modes of advective flow in karst

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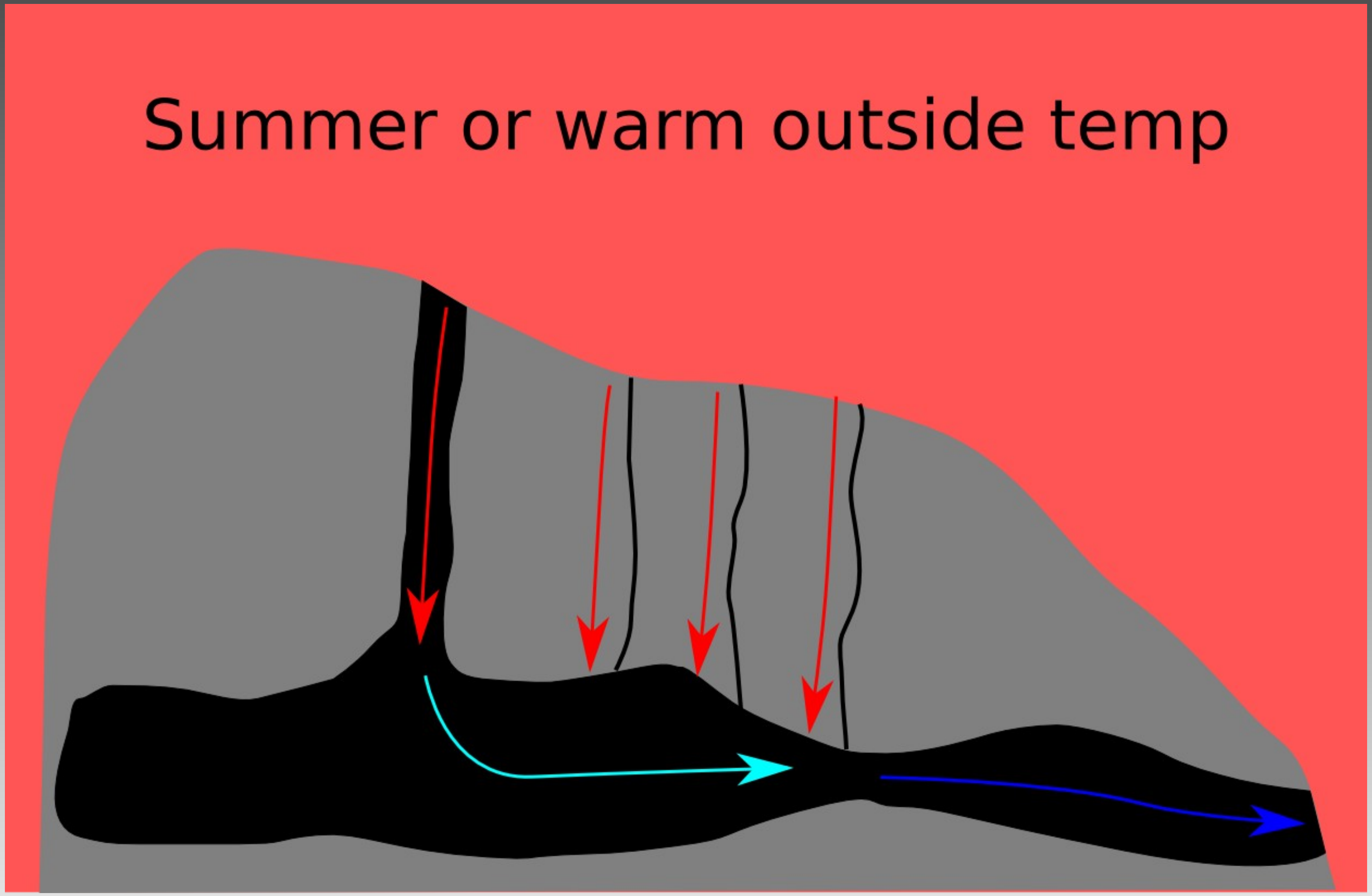


When outside temperature is **warmer** than cave temperature airflow is driven from ***higher*** to ***lower*** entrances

Advective flow dominates in caves, but what about fractures?

Photo: Hazel Barton

Advection vs. Diffusion: The Peclet Number

$$\mathrm{Pe} = \frac{t_{diff}}{t_{adv}} = \frac{VL}{D}$$

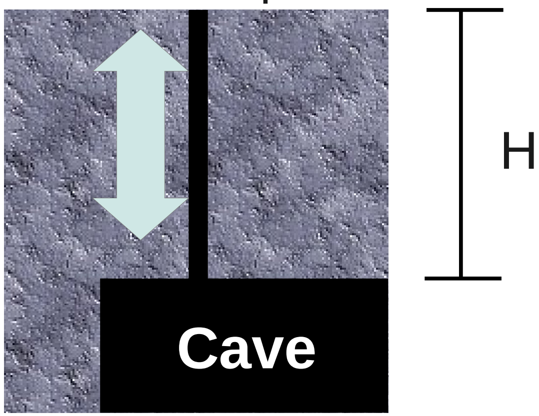
If Pe >> 1: advective

If Pe << 1: diffusive

For chimney-effect airflow, we get:

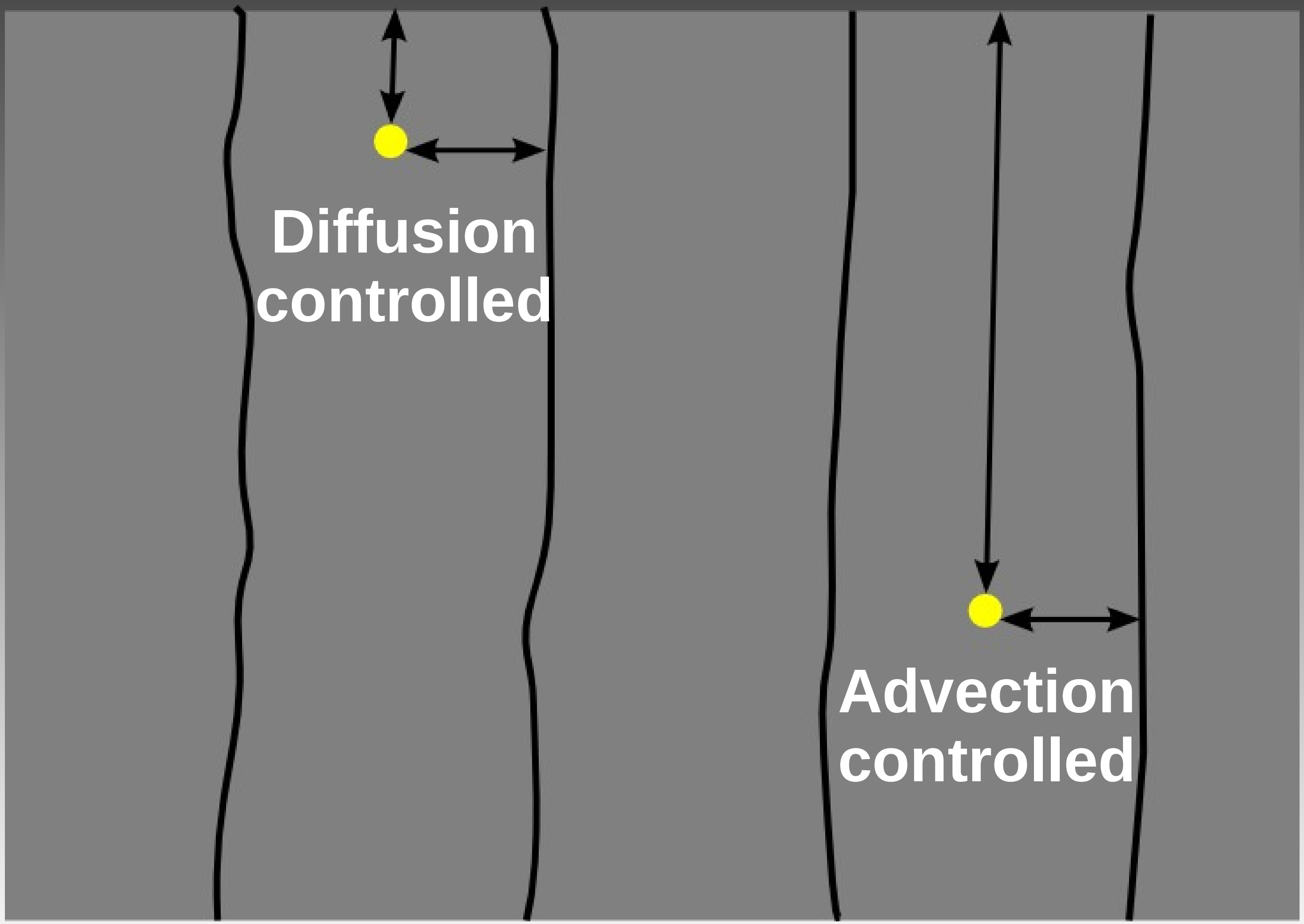
$$\mathrm{Pe} = \frac{a^2 g}{12 \nu D} \left(\frac{\Delta T}{T_{ext}} \right) H$$

a = fracture aperture



For a 1 mm fracture in a 10 m vadose zone, with a 3% temperature difference, $\mathrm{Pe} \sim 3\mathrm{x}10^5$!!!

Can advection really influence the bulk of the vadose zone or just isolated regions around caves and fractures?



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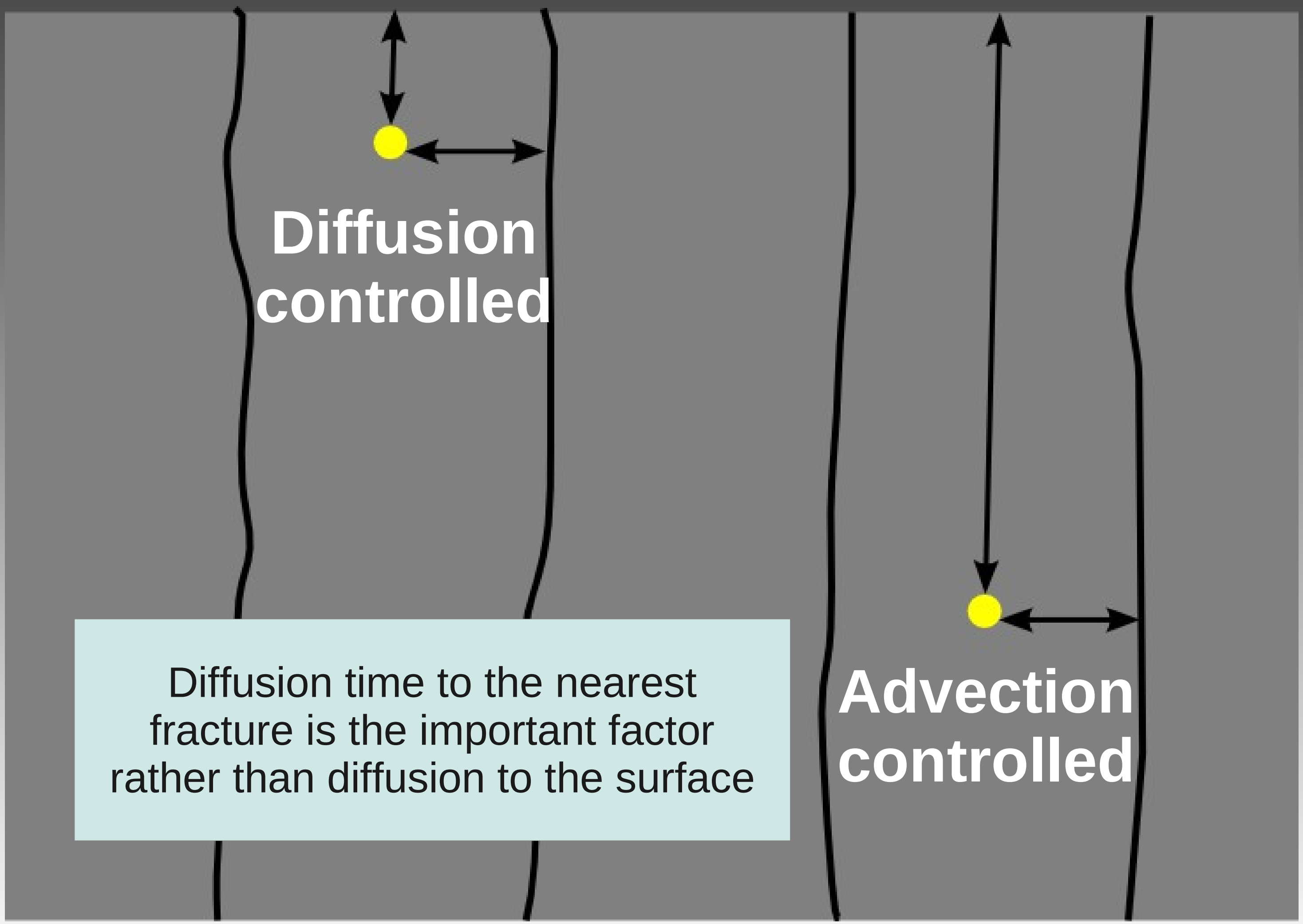
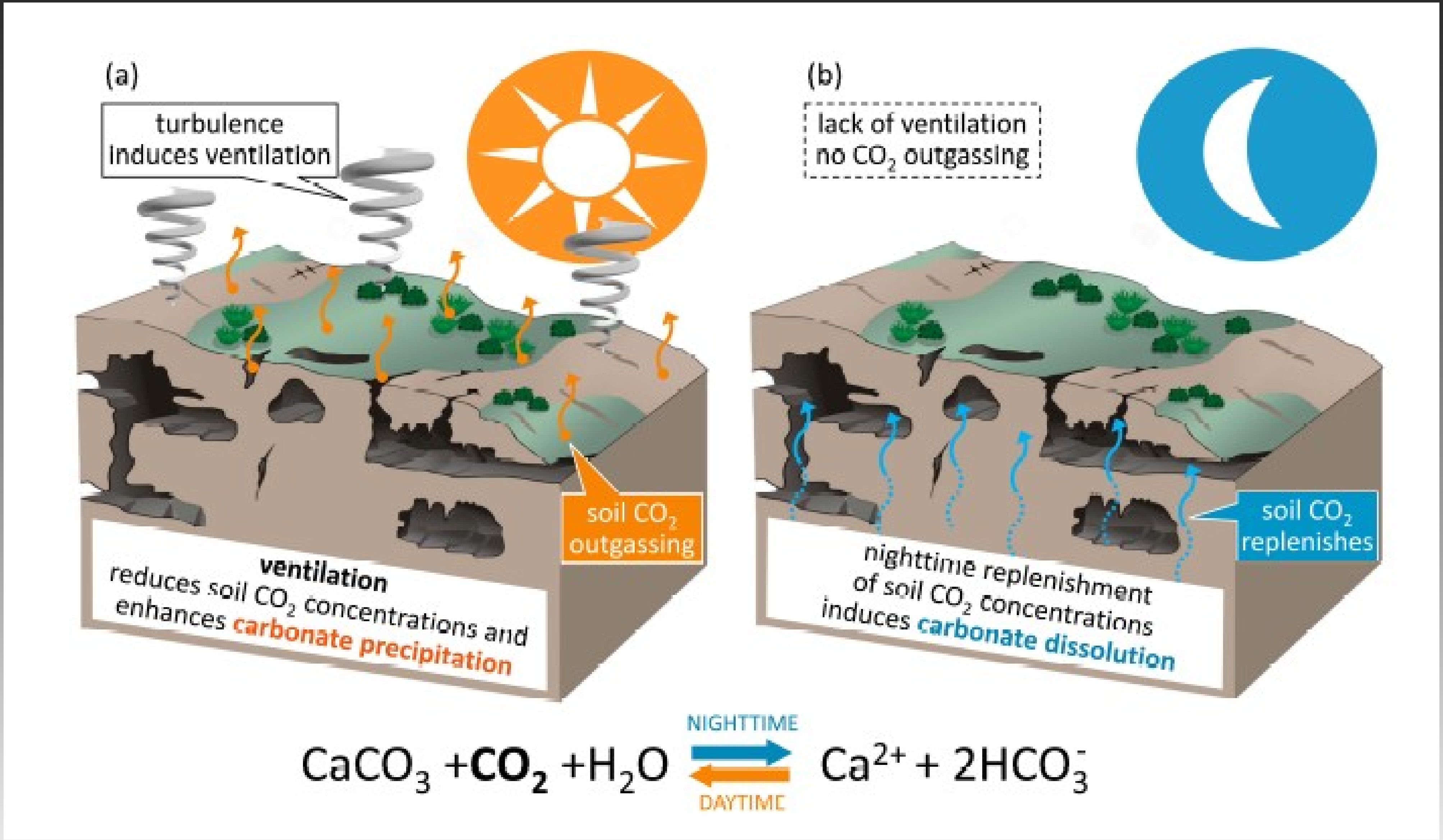


Wind-driven advective flows



Roland et al. (2013), Biogeosciences.

Loads of lengths

CO_2 production length scales:

Filtration
Decay of organic matter
Root depth distribution

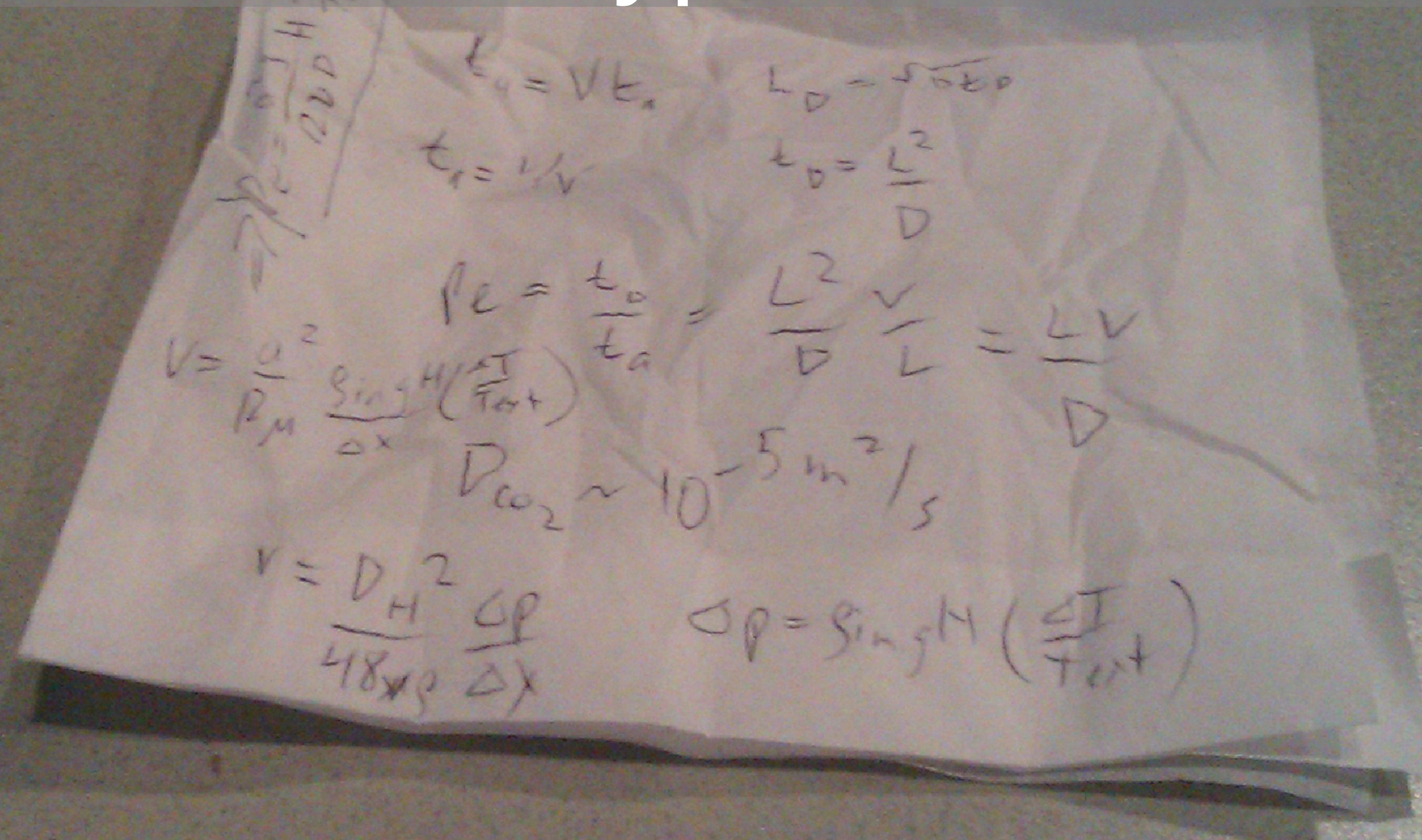
Air-water CO_2 exchange length scale:

Controls whether water equilibrates to air CO_2 or lags behind it with depth

Ratio of dissolution length scale to length scale over which CO_2 changes

Controls vertical distribution of dissolution

Simple models allow rapid generation of relevant questions and testable hypotheses



Temperature changes with depth from surface conduction (periodic forcing)

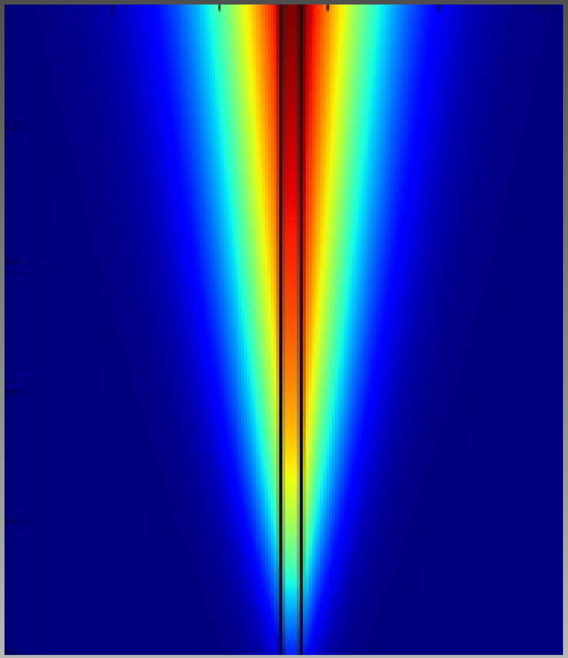
$$\text{skin depth} \sim \sqrt{\frac{\alpha_r T}{\pi}}$$

For diurnal variations: ~10 cm

For annual variations: ~3 m

Propagation of periodic thermal signals through advection down fractures

A periodic change in recharge water temperature leads to a thermal penetration length given by



$$\lambda_{\mathrm{T,sin}} = \frac{V \Psi D_{\mathrm{H}}}{4} \sqrt{\frac{2}{\alpha_{\mathrm{r}} \omega}}.$$

where

$$\Psi = \frac{\rho_w c_{p,w}}{\rho_r c_{p,r}}$$

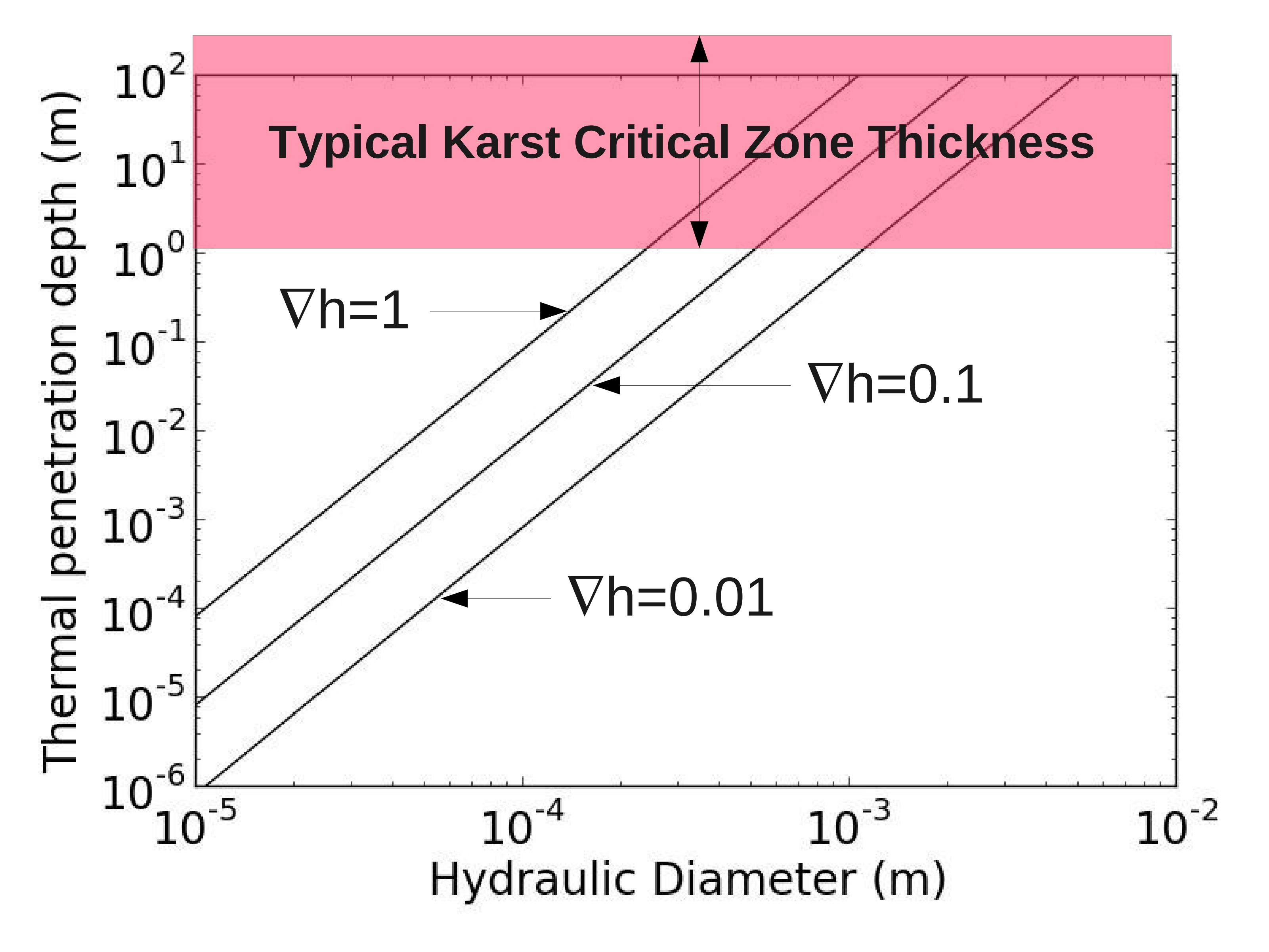
V is the flow velocity, D_H is the hydraulic diameter,

α_r is the thermal diffusivity of rock,

and ω is the angular frequency of temperature variations.

Luhmann et al. (in prep)

Propagation of periodic thermal signals through advection down fractures



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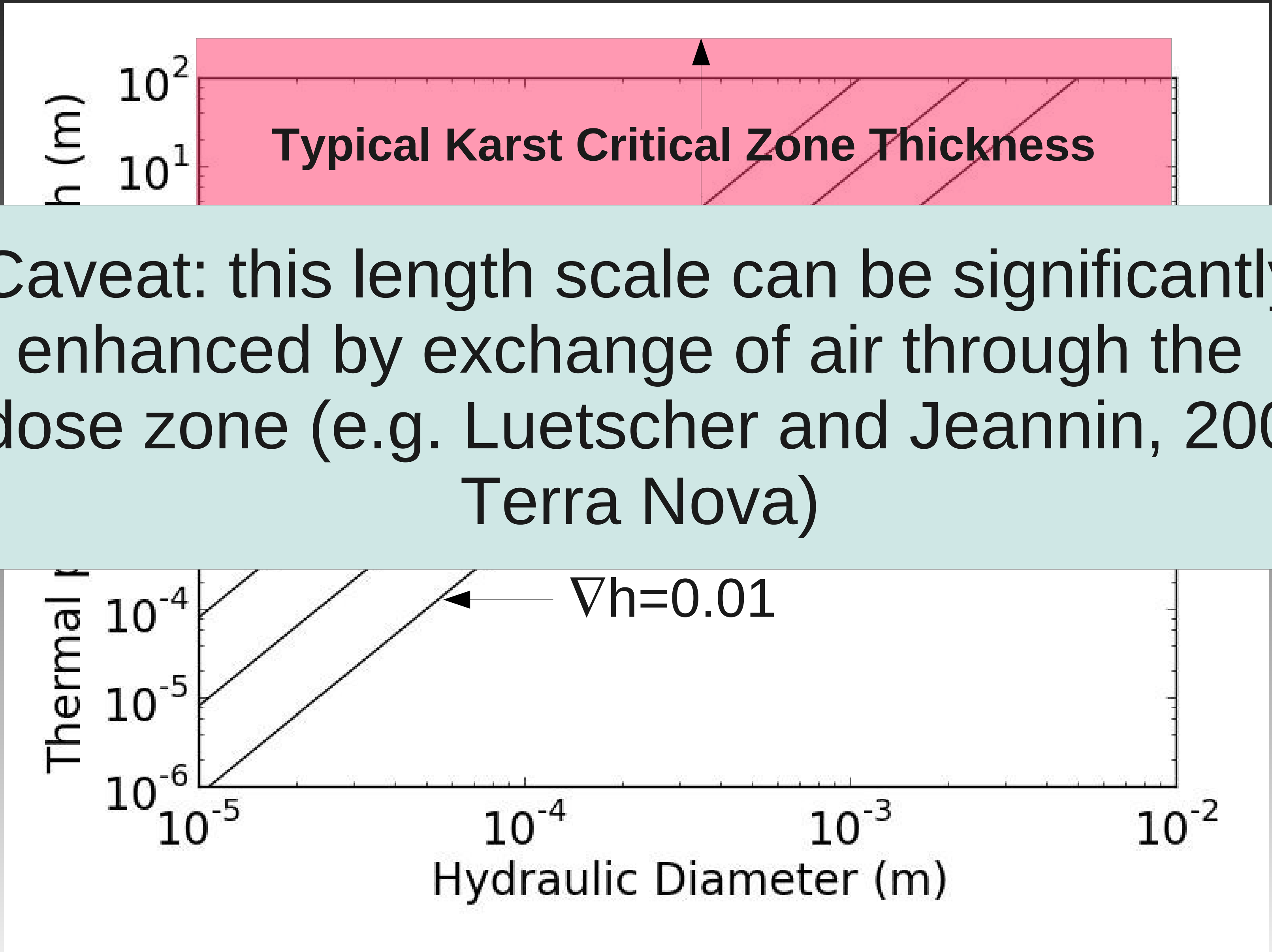
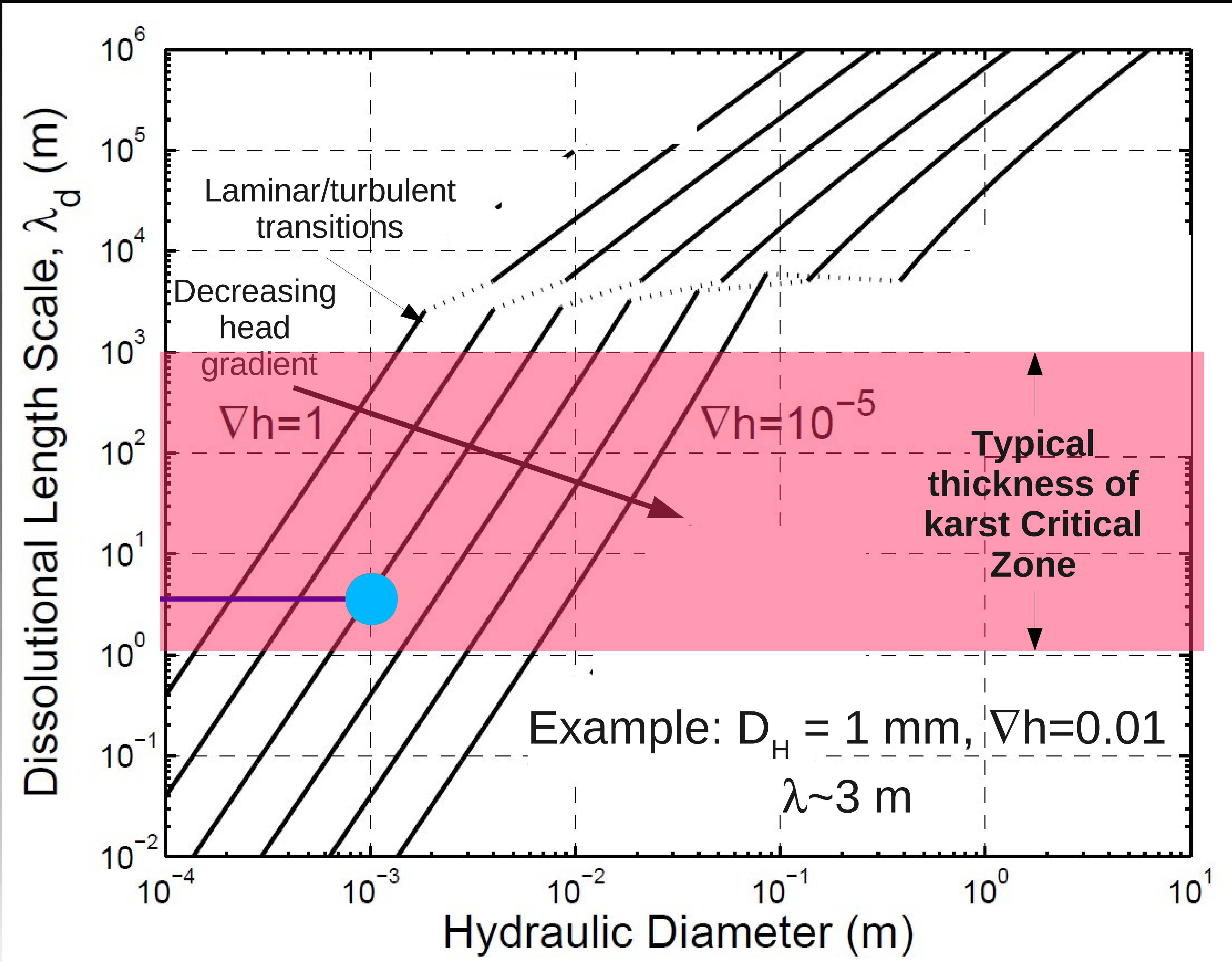
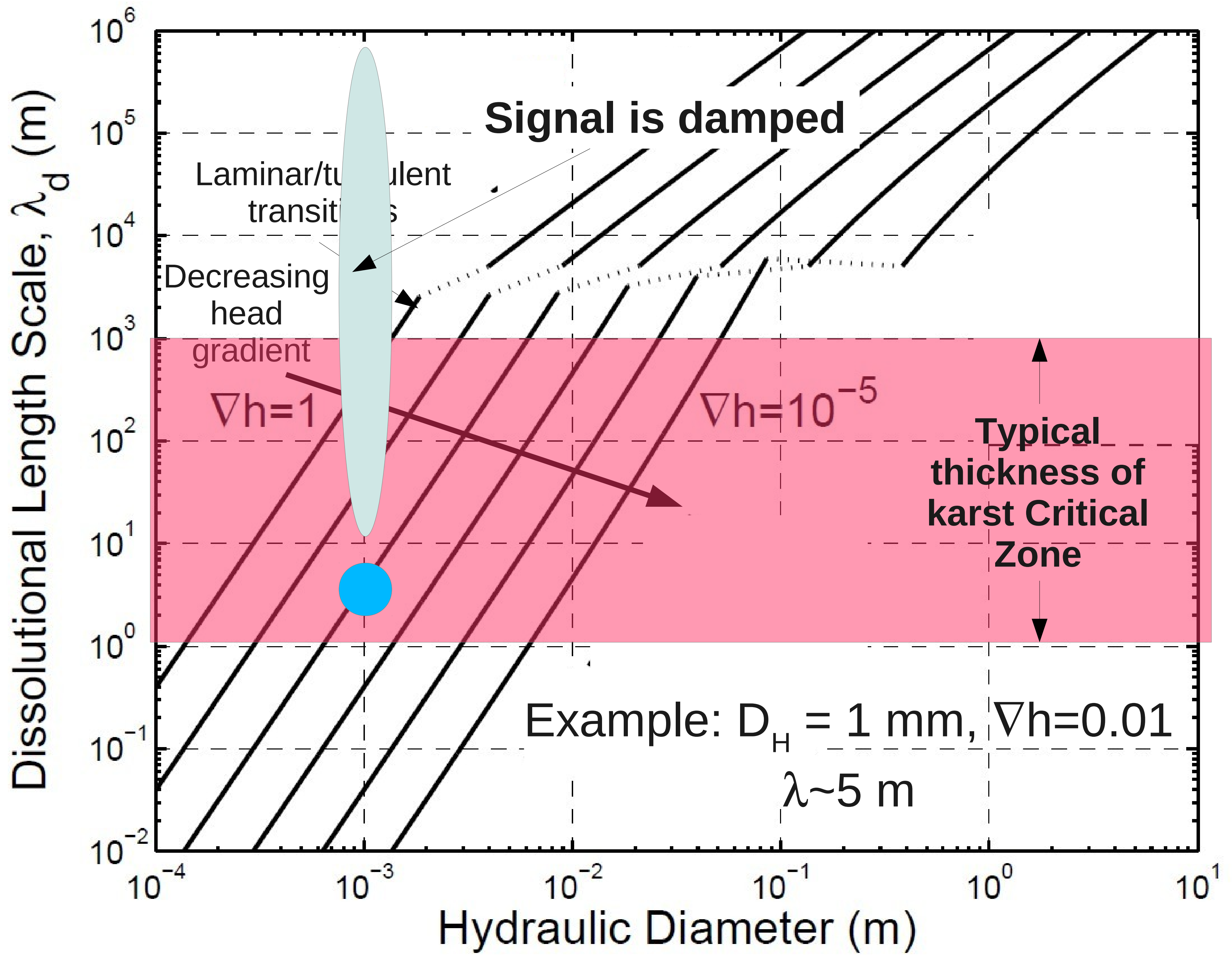


Caveat: this length scale can be significantly enhanced by exchange of air through the vadose zone (e.g. Luetscher and Jeannin, 2004, Terra Nova)



(Covington et al., 2012, J. Geophys. Res.)



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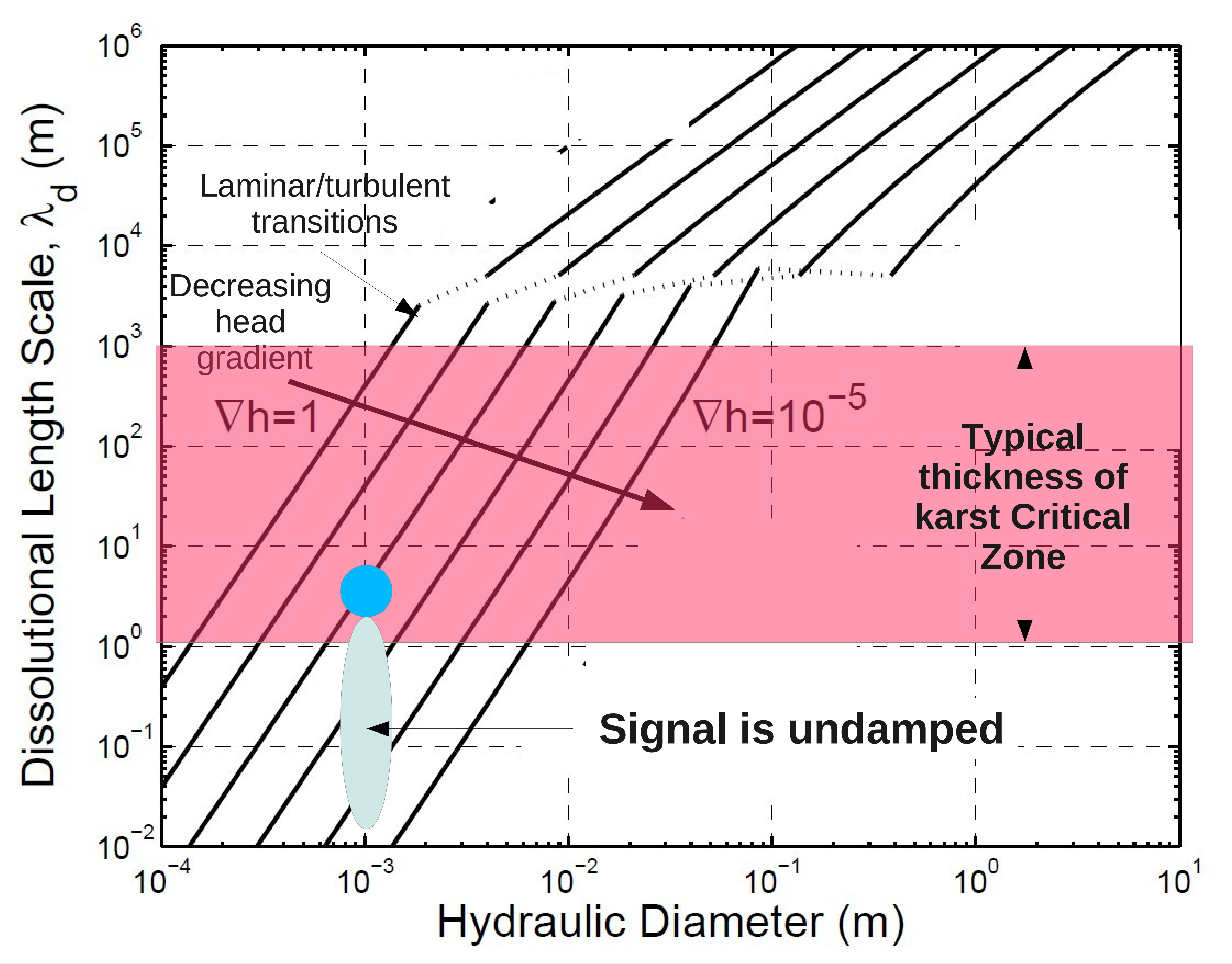

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