Generating Aquifer Specific Storage Properties from Groundwater Responses to Seismic Rayleigh Waves

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Abstract

A direct comparison between measurable groundwater fluctuations induced by seismic waves from large global earthquakes has previously been shown to be useful for evaluating specific storage of aquifers. However, most groundwater data loggers are set to record measurements several orders of magnitude too slow for comprehensive comparison with seismic shaking. A new computation procedure has been developed to deal with this sparse amount of water level data relative to seismological data. The method is applicable for water level data collected in a particular well for which a good estimate of aquifer transmissivity is available. The method demonstrates that water level deflection measurements during a single earthquake, if normalized to an appropriately filtered power spectrum of the associated Rayleigh wave motion, can provide a rough but unbiased estimate of the aquifer specific storage. Given a sufficient number of appropriate water level measurements during the passage of Rayleigh waves, specific storage can be computed to a similar accuracy as for continuous water level data. As a result of calculating the Rayleigh wave spectrum independently for each water level measurement during earthquakes, results from multiple earthquakes can be superposed to ensure a low computational error.

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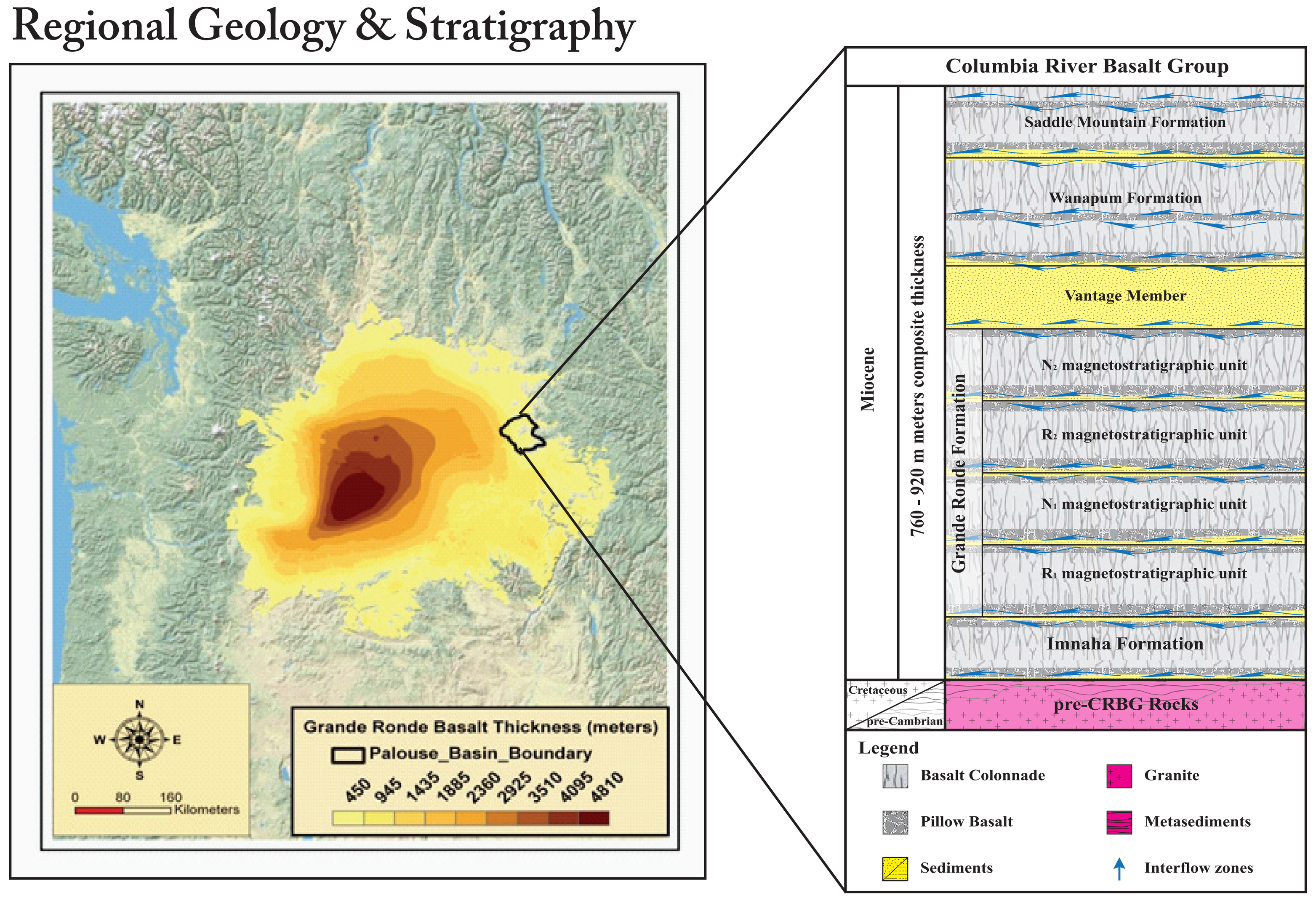


Figure 1: Areal extent of the Grande Ronde Formation and thickness (Burns, 2011) with Palouse Groundwater Basin boundary in black. Palouse Groundwater Basin stratigraphy with weathered flow tops acting as aquifer interflow zones.

Background

Seismological theory predicts that while a seismic Rayleigh wave (LR) of wavelength λ_k is passing, the relation between the vertical ground displacement and subsurface dilatation within a few hundred meters of the earth's surface is given by:

$$\Delta_k = -1.836\, \pi\, w_k / \lambda_k \qquad [1]$$

where Δ_k is the amplitude of the dilatation, and w_k is the amplitude of vertical displacement of the LR of wavelength λ_k>>z where z is the depth of the aquifer (Cooper et al., 1965; Stein and Wysession, 2003; Shih, 2009).

For a uniformly pourous confined aquifer, dilatation (the change in aquifer volume per unit volume) can be expressed in terms of specific storage (S_s) and water level change in an open borehole:

$$\Delta_k = -S_s\, H_k / R_k. \qquad [2]$$

H_k is the amplitude of the water level oscillation and R_k is the borehole amplification factor (Cooper et al., 1965). The borehole response can be estimated using the following formula (Cooper et al.,1965):

$$R_k = [\, (1 - \{\pi r^2 / T\tau\} \,\mathrm{Kei}\,\alpha - 4\pi^2 H_e / \tau^2 g)^2 + (\{\pi r^2 / T\tau\}\,\mathrm{Ker}\,\alpha)^2\,]^{-1/2} \qquad [3]$$

where $\alpha = r\,(\omega S_s b / T)^{1/2}$, r is the radius of the borehole, S_s is specific storage, b is the screened aquifer thickness, T is transmissivity, τ is wave period, ω is angular frequency of the wave, H_e is the effective height of the water column, and g is the gravitational acceleration. Ker and Kei are Kelvin functions of the second kind of order zero (eg http://keisan.casio.com).

Combining equations (1) and (2), one obtains, in theory, a connection between the water level oscillation in the borehole and the Rayleigh wave displacement on the surface:

$$H_k = (1 / S_s)\,(5.77\, R_k\, w_k / \lambda_k) \qquad [4]$$

With the exception of S_s, all the variables in the above equation are functions of frequency (as indicated by the subscript k). In practice, spectral methods need to be employed to transform the LR displacements (Figure 2A) and the water level fluctuations (Figure 2B) into their constituent frequency components to get useful results. About fifteen moderately large earthquakes (magnitude 7+) occur each year, each producing very high amplitude surface waves for tens of minutes after the shocks. From a statistical viewpoint, these result in a significant amount of data with which to work even if the water levels are only measured every few minutes.

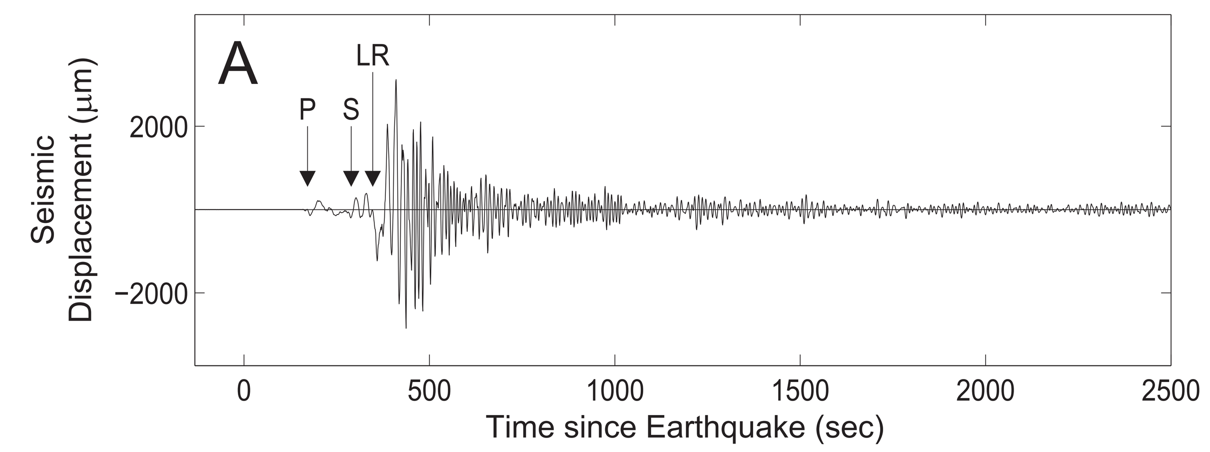
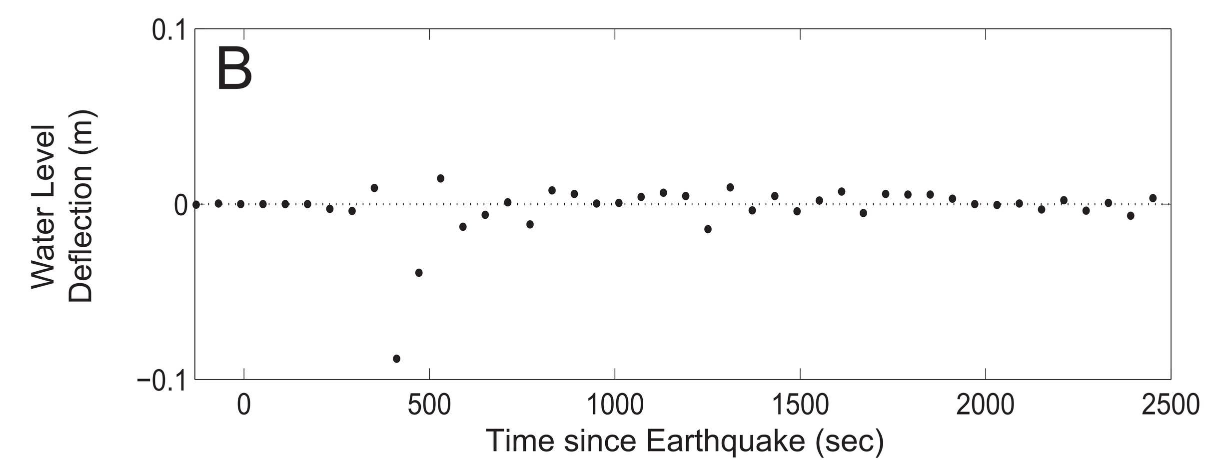


Figure 2: **A.** The M7.5 Haida Gwaii Earthquake. Vertical ground displacement at municipal well M9 in Moscow, Idaho based on regional seismograph station BRAN. The Rayleigh wave arrives at the time indicated by LR and continues across the record.



B. Water level changes driven by the Rayleigh wave as sampled measured at 1-min intervals by a Solinst Levelogger Gold ®.

Methodology

In terms of the discrete Fourier transform F, equation [4] can be written as follows, taking note that λ and R are dependent on frequency as indicated by the subscript k.

$$F\{H_k\} = (1/S_s)\,(5.77\, R_k / \lambda_k)\, F\{W_k\} \qquad [5]$$

Now the complex conjugate (*) of [5] is taken:

$$F\{H_k\}^* = (1/S_s)\,(5.77\, R_k / \lambda_k)\, F\{W_k\}^* \qquad [6]$$

Multiplying [5] times [6], and dividing by the number of samples N in the time sequences:

$$HH_k = (1/S_s^2)\,(33.3\, R_k^2 / \lambda_k^2)\, WW_k \qquad [7]$$

where: HH_k is the power spectra of the water level oscillations forced be the seismic displacements, defined by:

$$HH_k = F\{H_k\}\, F\{H_k\}^* / N \qquad [8]$$

and WW_k is simply the power spectra of w, defined by:

$$WW_k = F\{W_k\}\, F\{W_k\}^* / N \qquad [9]$$

It is interesting to note that the adjustment $(33.3\, R_k^2 / \lambda_k^2)$ created by the wavelength and the borehole response not only enhances oscillations near the resonant frequency of the borehole but also reduces the influence of the longer wavelength (and typically higher amplitude) Rayleigh waves on the water level oscillations.

Dividing both sides of [7] by N, and using Parseval's theorem, one finds that the mean squared water deflection can be predicted from the mean value of the Rayleigh wave displacement spectral density after adjustment for wavelength and borehole effects. That is,

$$E\{h^2\} = (1/S_s^2)\, E\{(33.3\, R_k^2 / \lambda_k^2)\, WW_k\} \qquad [10]$$

where: the expected value operator E represents an average value over the sample interval.

Rearranging [10], the term involving specific storage is given by the ratio of the mean squared water level deflection to the adjusted mean seismic power during the passage of the Rayleigh wave segment.

$$(1/S_s^2) = E\{h^2\} / E\{(33.3\, R_k^2 / \lambda_k^2)\, WW_k\} \qquad [11]$$

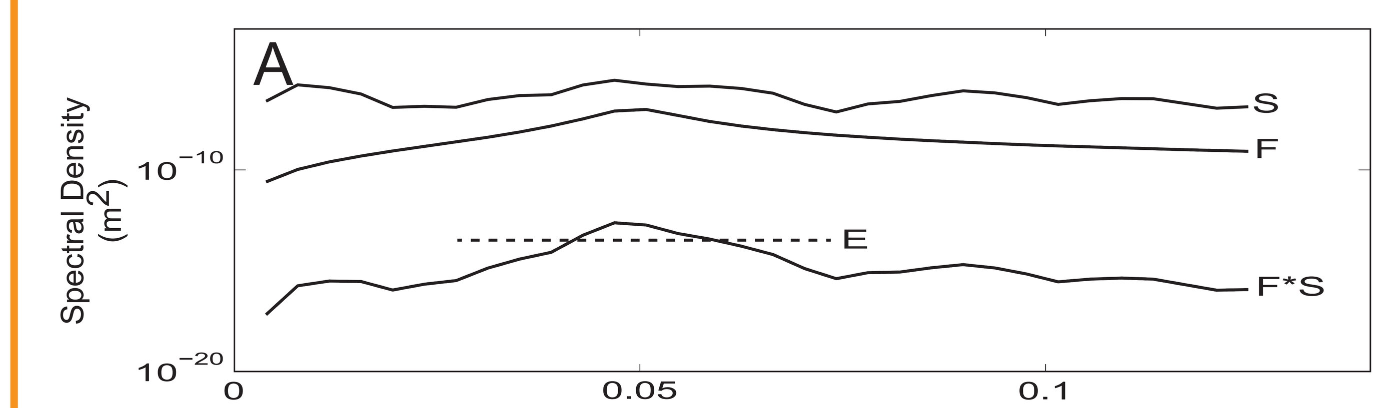
Because the water level data are sparse, for any given Rayleigh wave segment, $E\{h^2\}$ is not known. The only estimate available is the single instantaneous value h_i. Nonetheless, for each water level measurement available, the associated Rayleigh wave segment can be processed to get an independent estimate of $(1/S_s^2)$ by setting $E\{h^2\} = h_i^2$.

$$(1/S_s^2) = h_i^2 / E\{(33.3\, R_k^2 / \lambda_k^2)\, WW_k\} \qquad [12]$$

We used the Rayleigh wave segment from 75 s before to 75 s after each water level observation to compute the filtered power spectra (Fig. 3). A considerable number of water measurements and associated Rayleigh wave segments are required to get a reasonable estimate of $(1/S_s^2)$. We use the number (N') of water level measurements available to compute our final value of $(1/S_s^2)$, which is simply the arithmetic mean of our estimates.

The new estimate of specific storage is then the inverse square root of $(1/S_s^2)$. This value is then used to recalculate R_k and the entire procedure above is iteratively repeated until a final value of S_s is found that matches the value used in the calculation of R_k. The mean $1/S_s^2$ and 68% confidence intervals on the mean are shown in Fig. 4.

Our final estimate of S_s is $1.5 \times 10^{-6}\ \mathrm{m}^{-1}$. The standard error of this value is about 10%, a computational precision consistent with our simulation results for the case of 60 water level measurements.



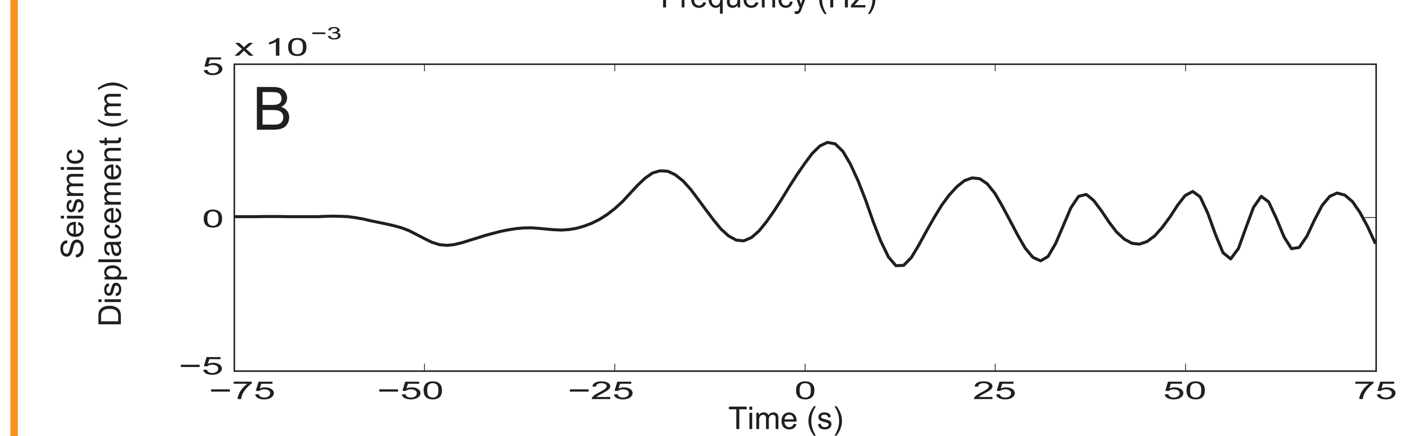


Figure 3: ***A.*** *The plot designated as S is the power spectrum WW_k of the Rayleigh wave segment. Plot F is the filter $(33.3\, R_k^2 / \lambda_k^2)$ used to adjust the spectrum for borehole amplification and wavelength. Plot F * S is the filtered power spectrum. Level E is at the expected value (mean) of the filtered power spectrum.* ***B.*** *The 150-s segment of the Rayleigh wave (Fig. 2) for the Haida Gwaii Earthquake centered about the water level measurement 410 s after the origin time. Note that the seismic data in the segment shown have been filtered to remove noise above 0.125 Hz whereas the data shown in Fig. 1 have not.*

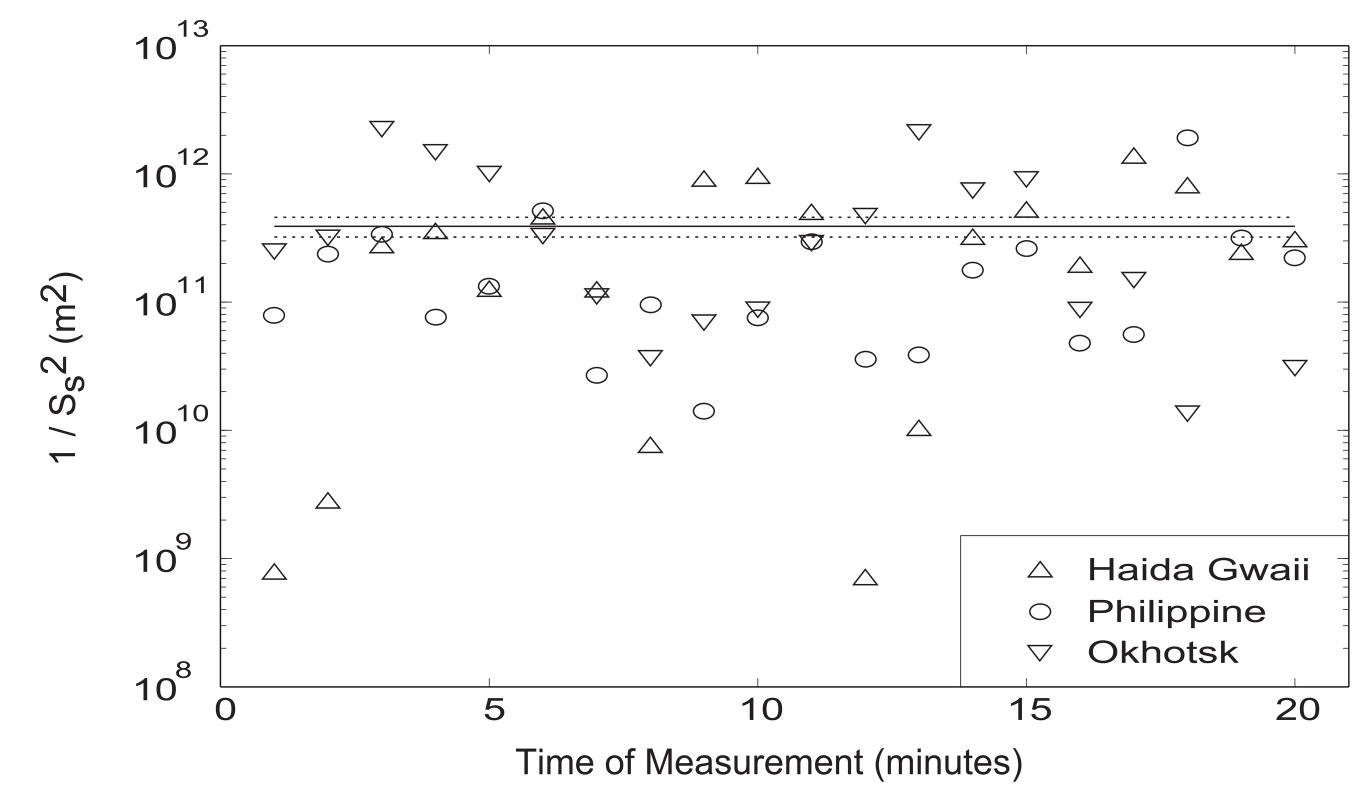


Figure 4: Individual estimates of $(1/S_s^2)$ based on instantaneous water fluctuation measurements made at the time indicated after the initial arrival of the Rayleigh wave train for each seismic event. The mean (solid line) and its 68% confidence intervals on the mean (dashed lines) are also shown.

Conclusions

1. This procedure requires water level data that is logistically simple to collect and the high quality regional seismological data that are now freely available in digital form for most areas of the world.

2. The application of this procedure is only for water wells in confined aquifers of known transmissivity which behave as predicted by seismological and well hydraulic theory for uniformly porous media.

3. An advantage of the method is that the entire aquifer is stressed almost simultaneously by the long wavelength Rayleigh waves, quite unlike the localized stress field associated with most pump tests.

4. The precision of the algorithm is strongly dependent on the number of water level measurements available during the passage of Rayleigh waves.

5. Specific storage results from multiple earthquakes can be combined to ensure a low computational error.

Acknowledgements

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