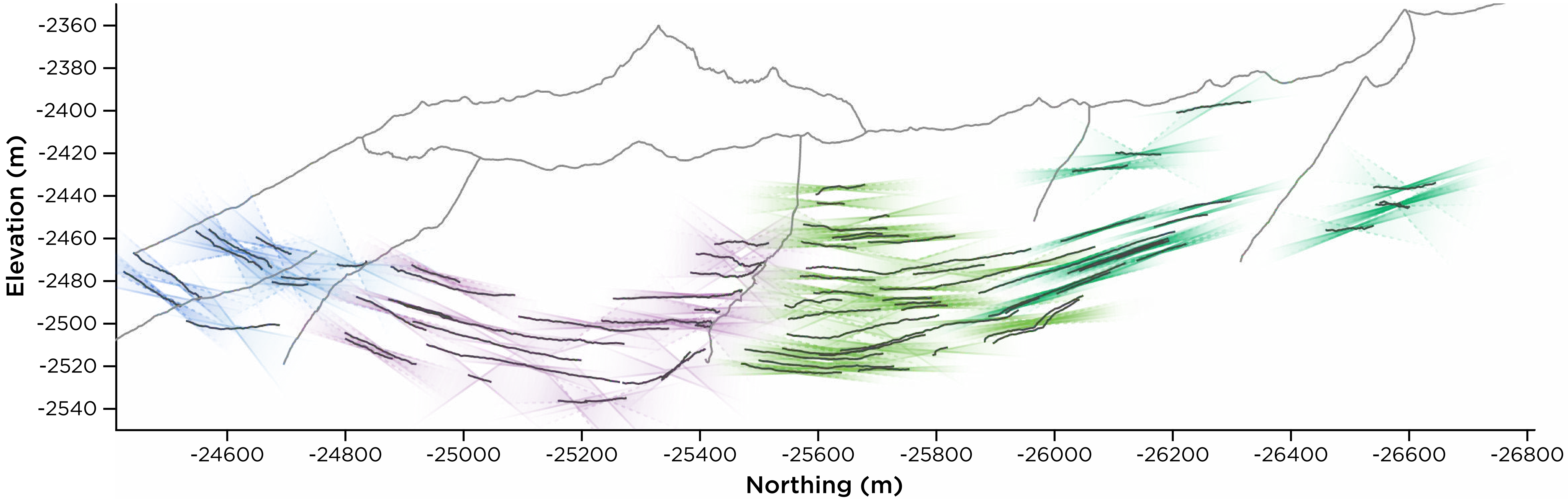
A statistical framework for determining remotely-sensed geological surface orientations and their error distributions

Daven P. Quinn[1,2*] and Bethany L. Ehlmann[2,3]

1 *University of Wisconsin—Madison* 2 *Caltech (* at the time of work)* 3 *Caltech–Jet Propulsion Lab*



Preprint manuscript: https://eartharxiv.org/4enzu

Motivation: Mars stratigraphy and structure

HiRISE *imagery (0.25 m/pixel) draped on photogrammetric elevation model (1 m per pixel)*

Motivation: orientations to test depositional mechanisms

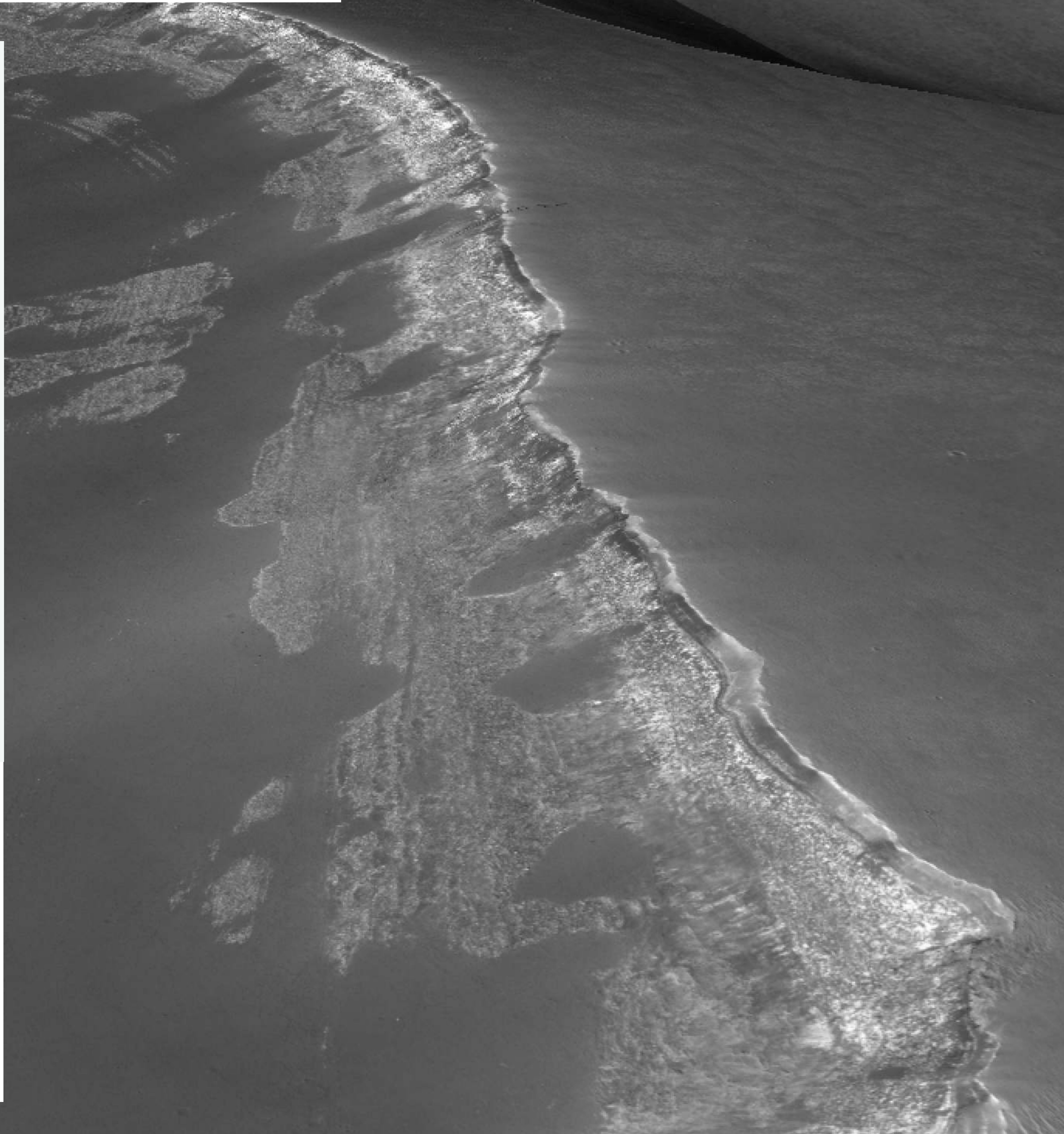
Bedding trace (323 m long)

- High correlation coefficient
- Non-unique angular orientation

Motivation: high-quality bedding orientations in general

The same problem operates for remote sensing of stratigraphies on Earth — this is particularly important in light of new tool such as UAVs.

Can we create general tools to and visualize the uncertainty of planar measurements?

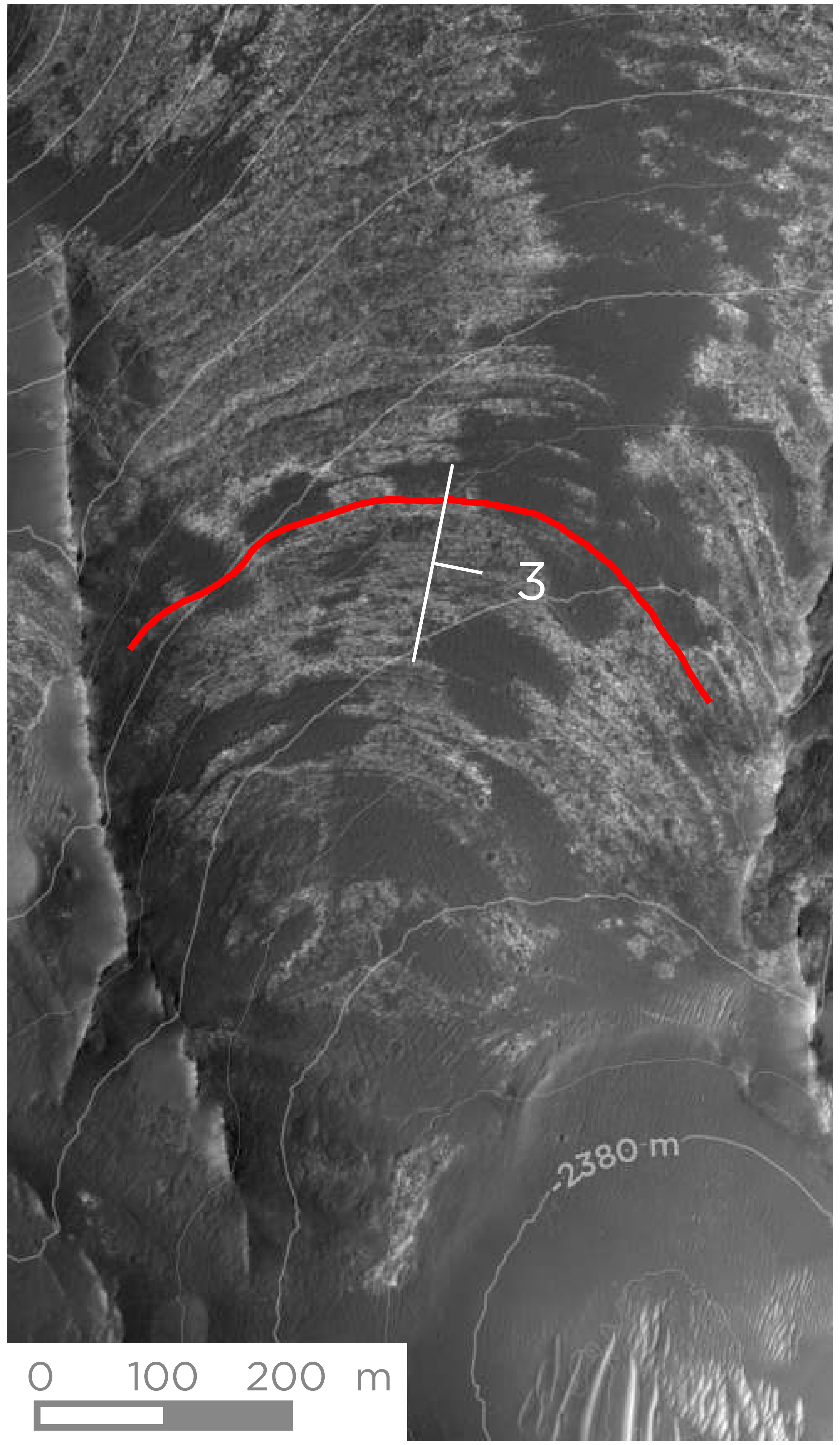


Orientation error structure

Views of or

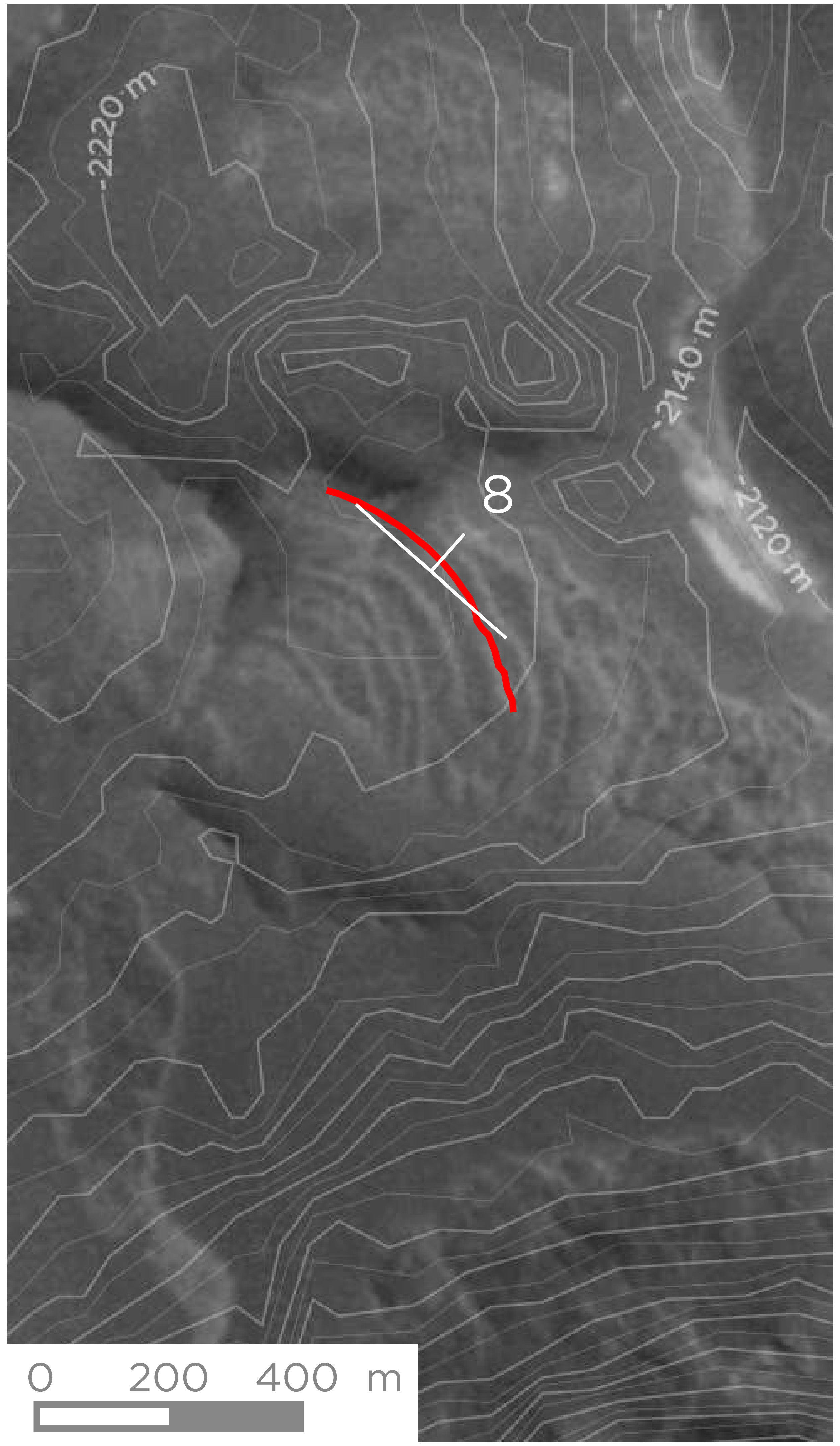
Curved exposure

Well-resolved orientation



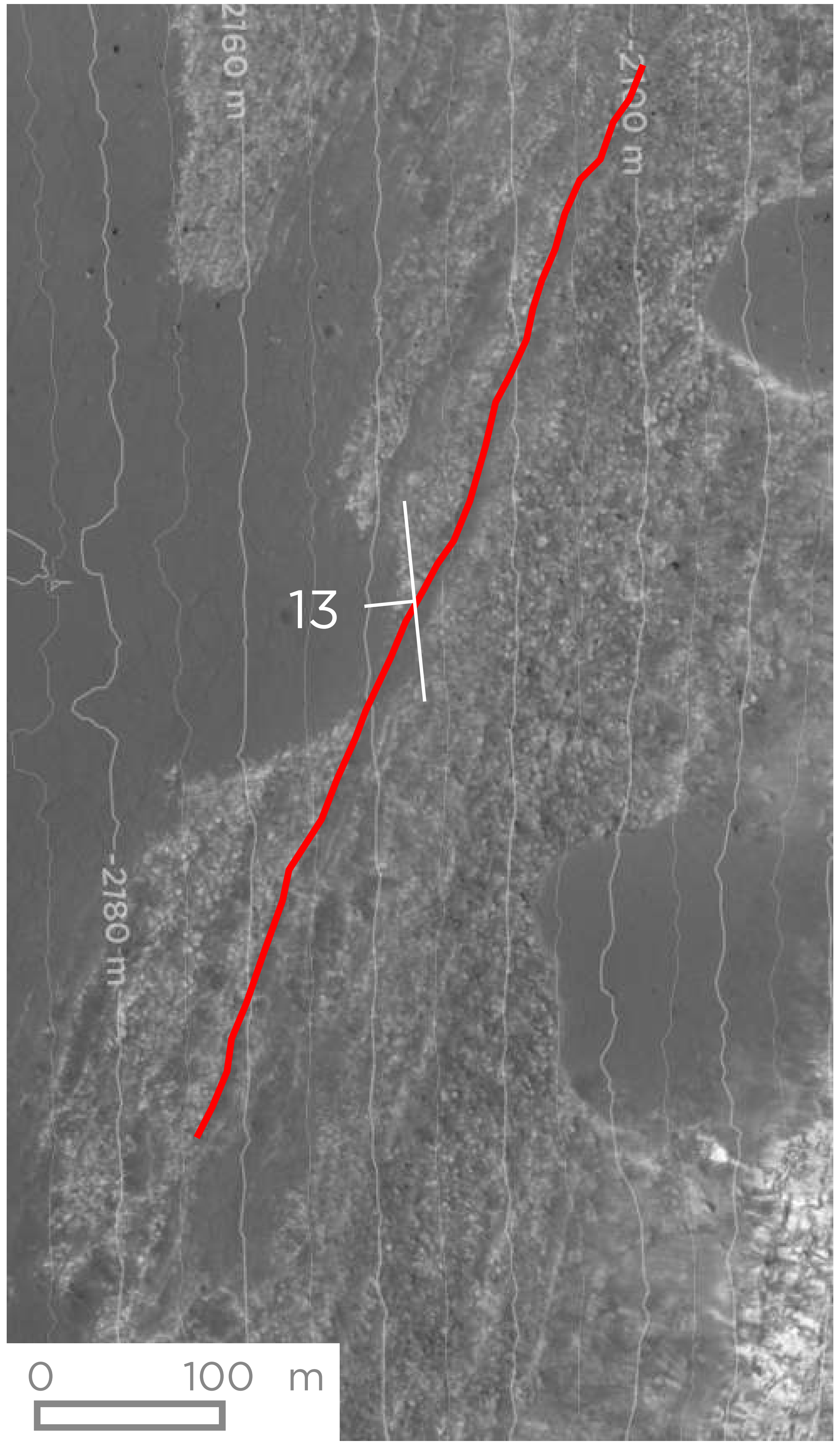
Low-resolution data

Larger orientation errors



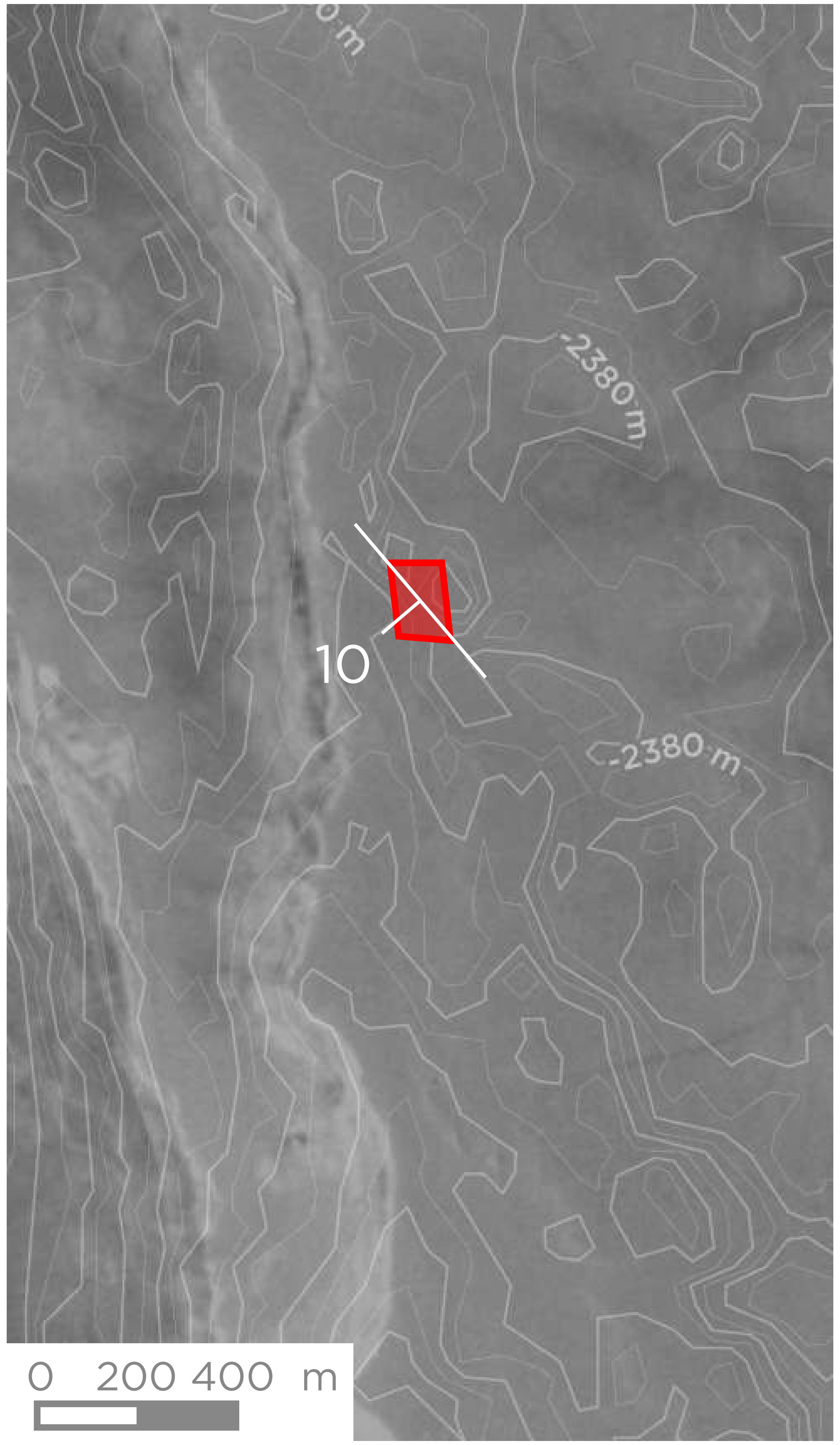
Planar hillslope

Constrained in one direction



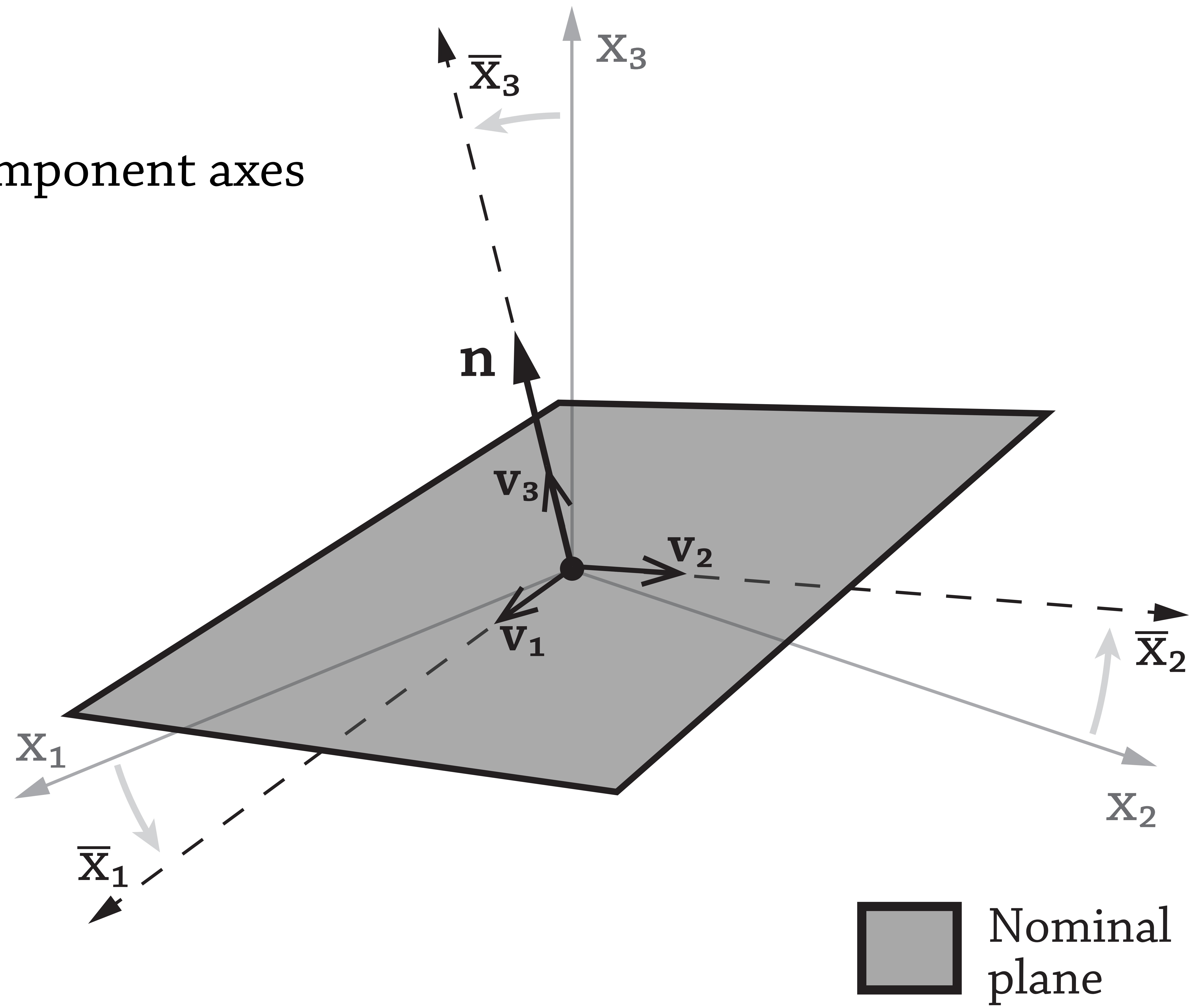
Poor-quality DEM

Garbage in, garbage out!



Conceptual framework for orientation errors to a plane
Nominal plane: rotated from global coordinate basis

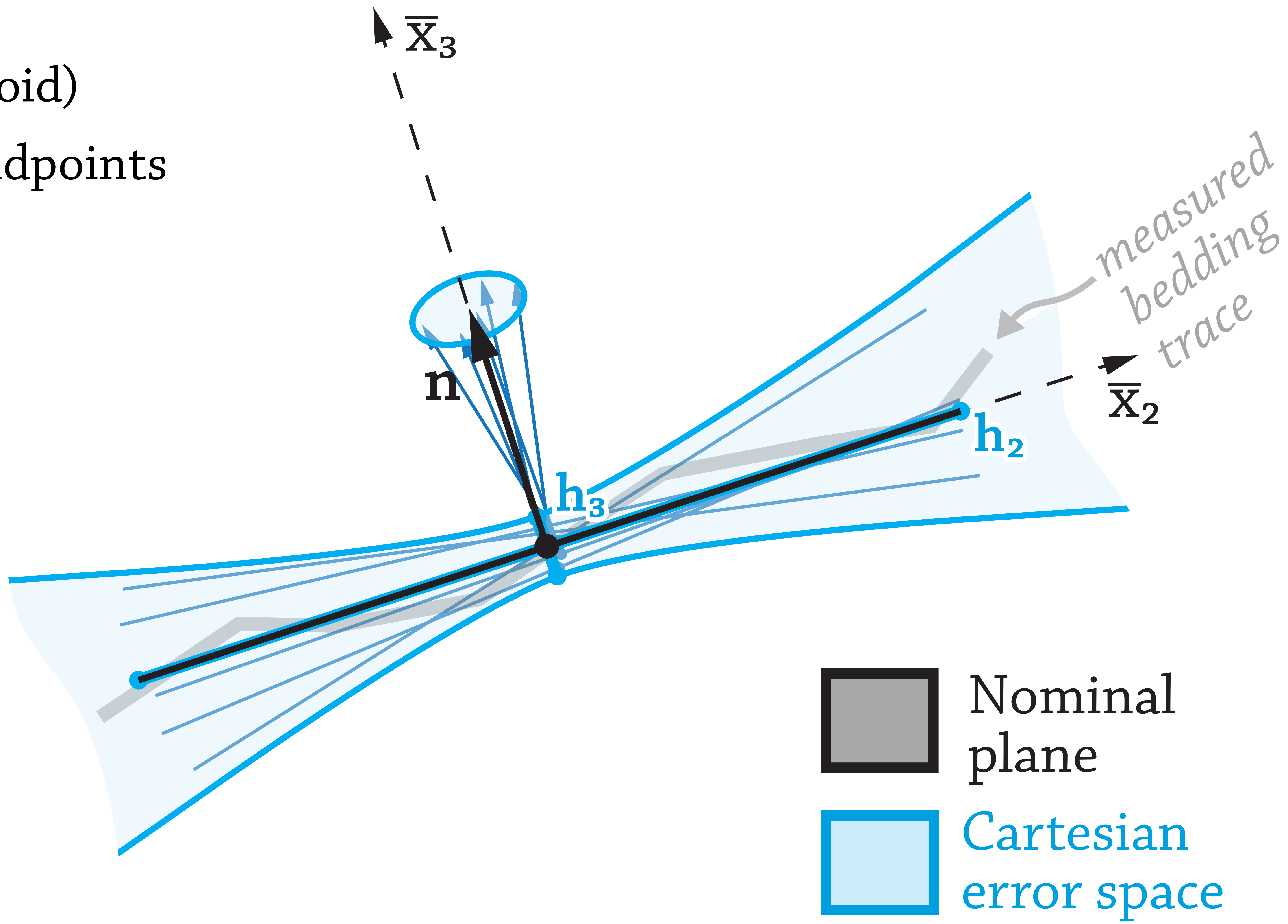
- Coordinate basis **x**: global Cartesian plane
- Coordinate basis $\overline{\mathbf{x}}$: aligned with principal component axes
- Normal vector **n** = $\overline{\mathbf{x}}_3$: perpendicular to plane



Conceptual framework for orientation errors to a plane

Error space: quadric *(generalized conic)* surfaces enclosing nominal plane

- Bundle of all possible planes (hyperboloid)
- Bundle of all possible normal vector endpoints (ellipsoid)



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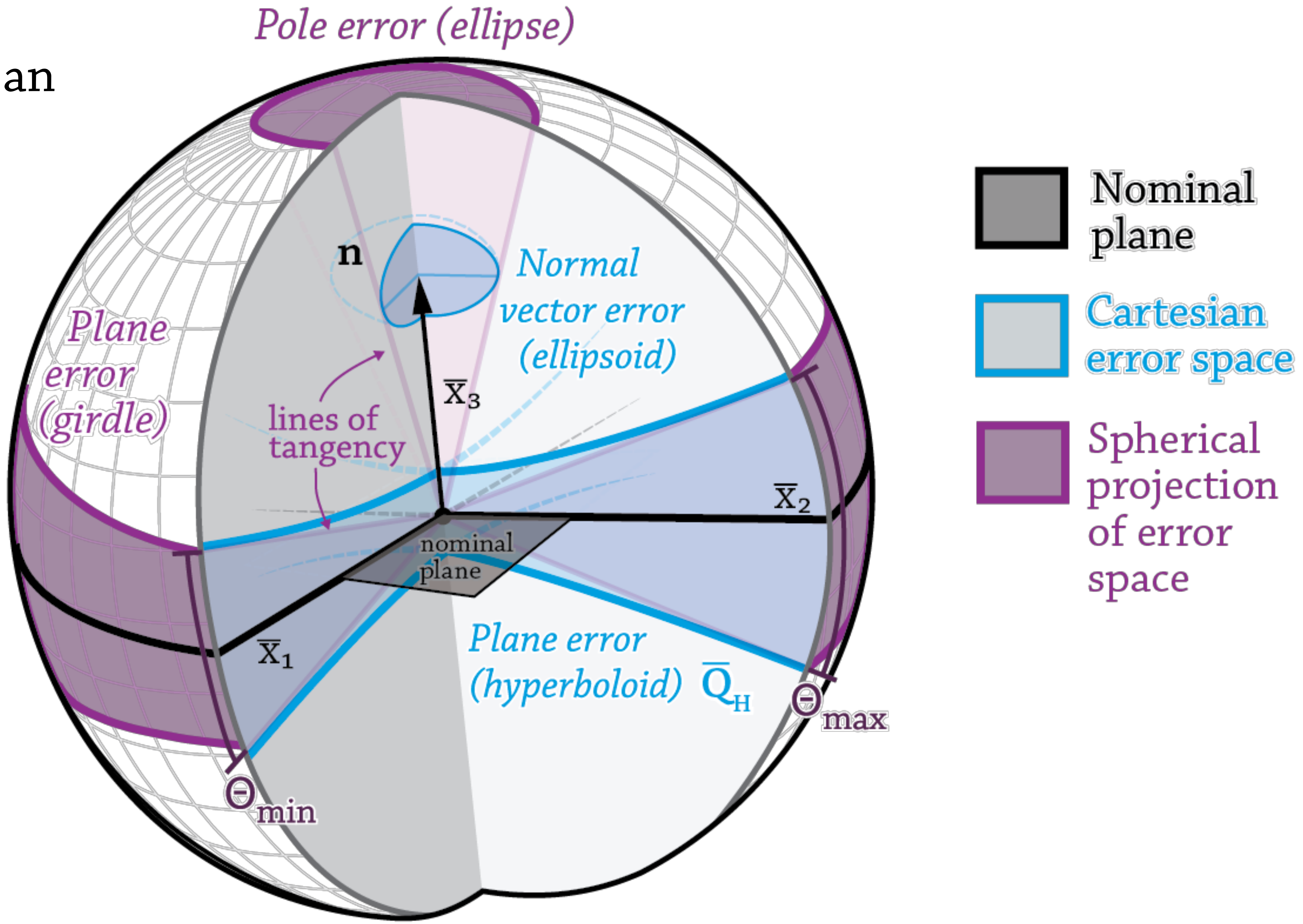
Conceptual framework for orientation errors to a plane

Error space: projection of Cartesian quadrics to spherical coordinates

- Angular errors can be parameterized as an ellipse or girdle on the unit sphere



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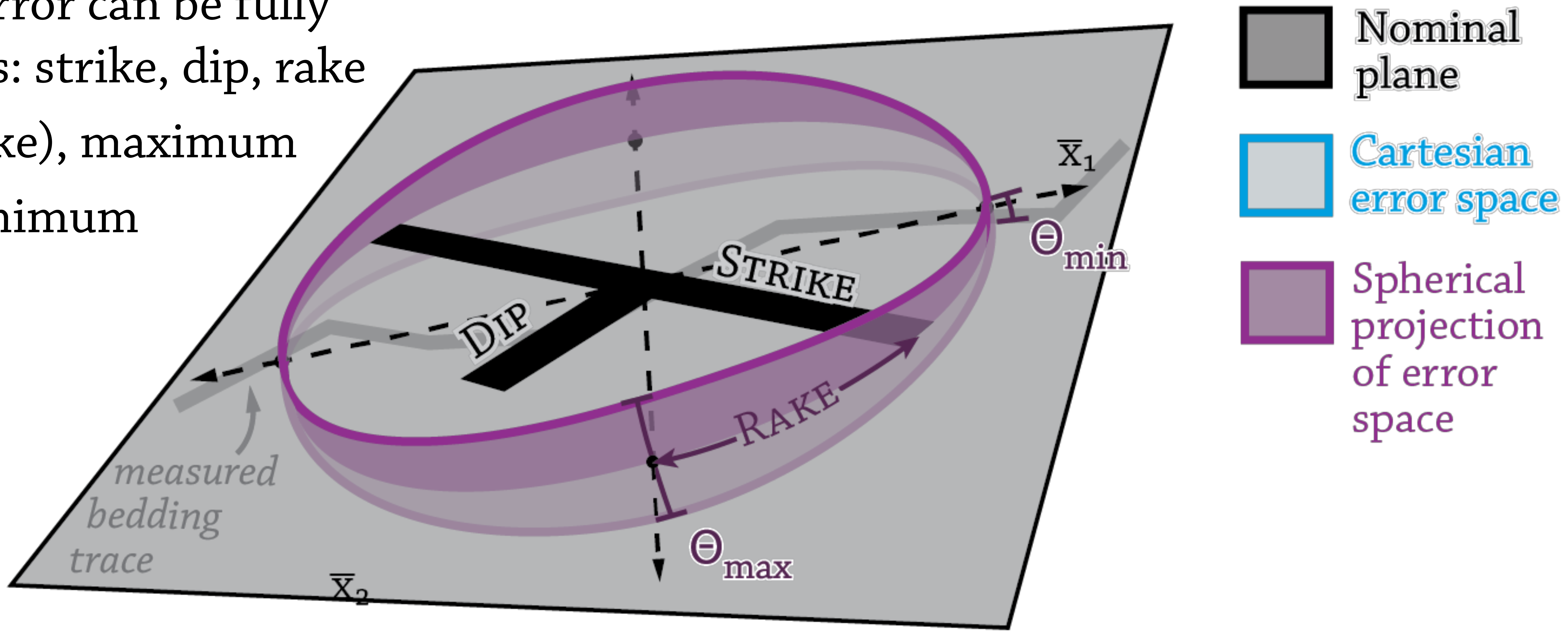
Conceptual framework for orientation errors to a plane
Error space: projection of Cartesian quadrics to spherical coordinates

- Angular errors can be parameterized as an ellipse or girdle on the unit sphere
- A planar orientation with error can be fully represented with five angles: strike, dip, rake (distance from Θ_{max} to strike), maximum angular error Θ_{max}, and minimum angular error Θ_{max}



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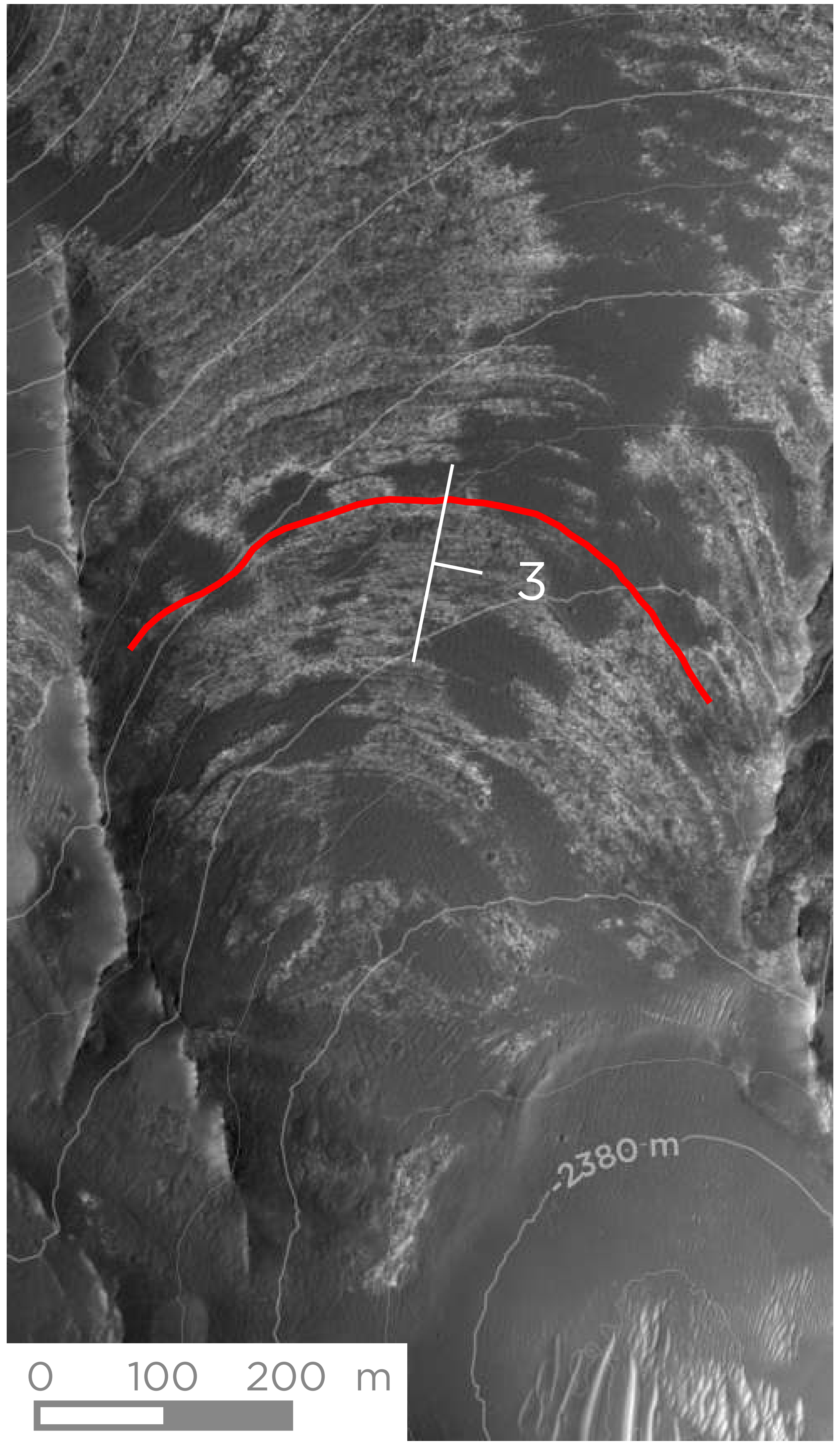
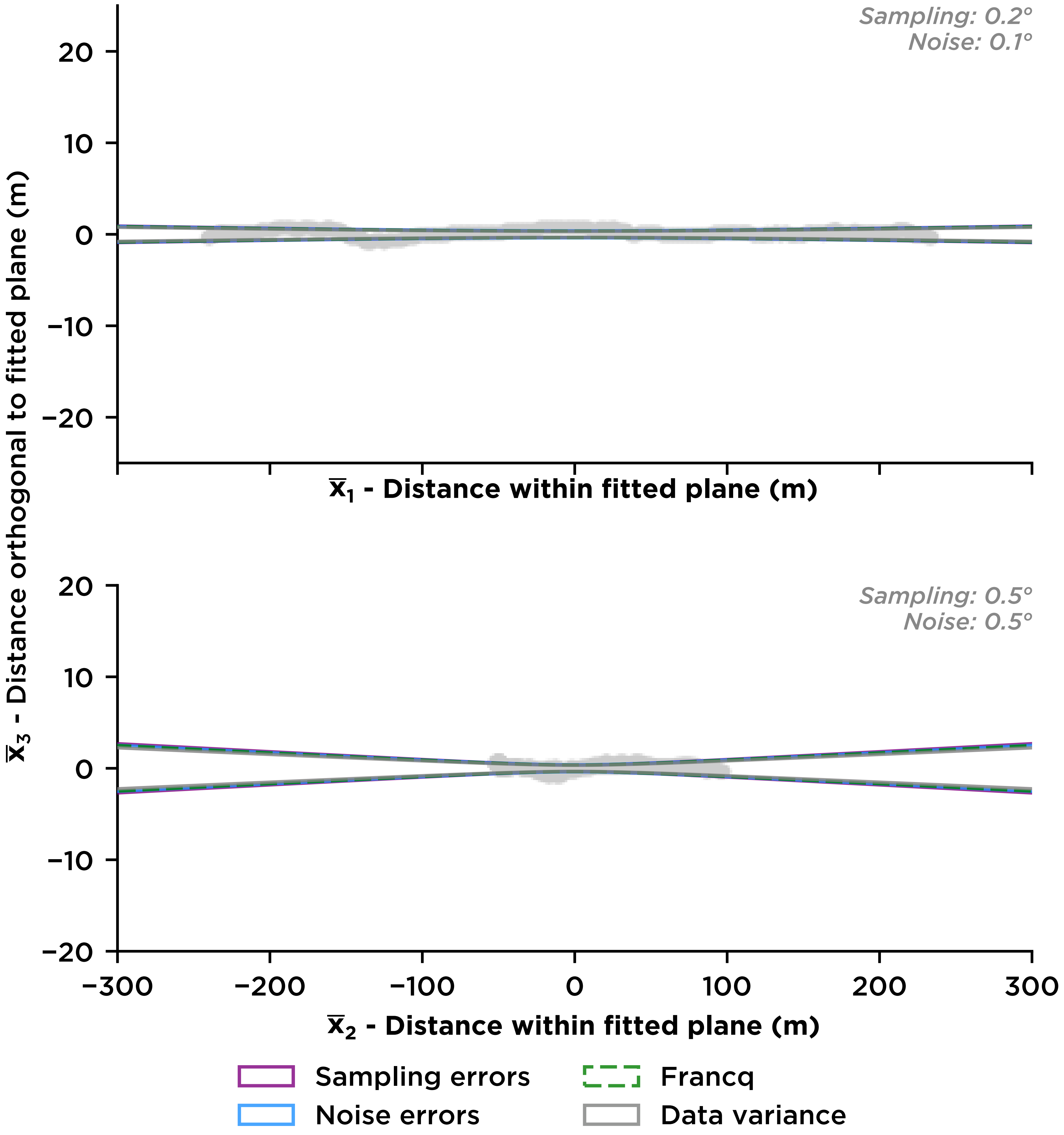
Orientation error structure: complimentary views of error space

Curved exposure

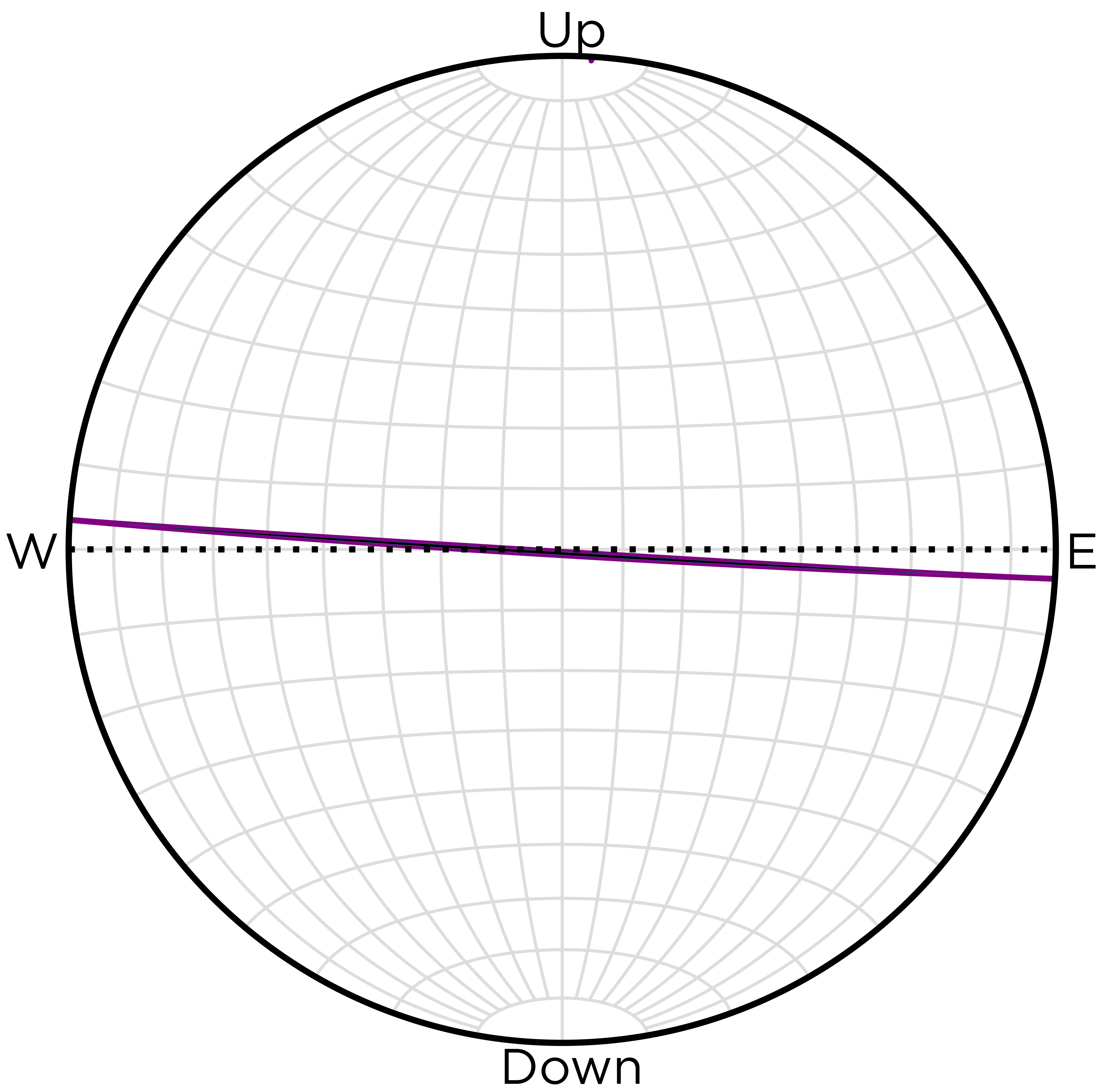
Well-resolved orientation



Aligned with principal component axes (shows variance of data)



Projected into spherical coordinates (shows entire error space)



Upper-hemisphere, equal-area rotated stereonet

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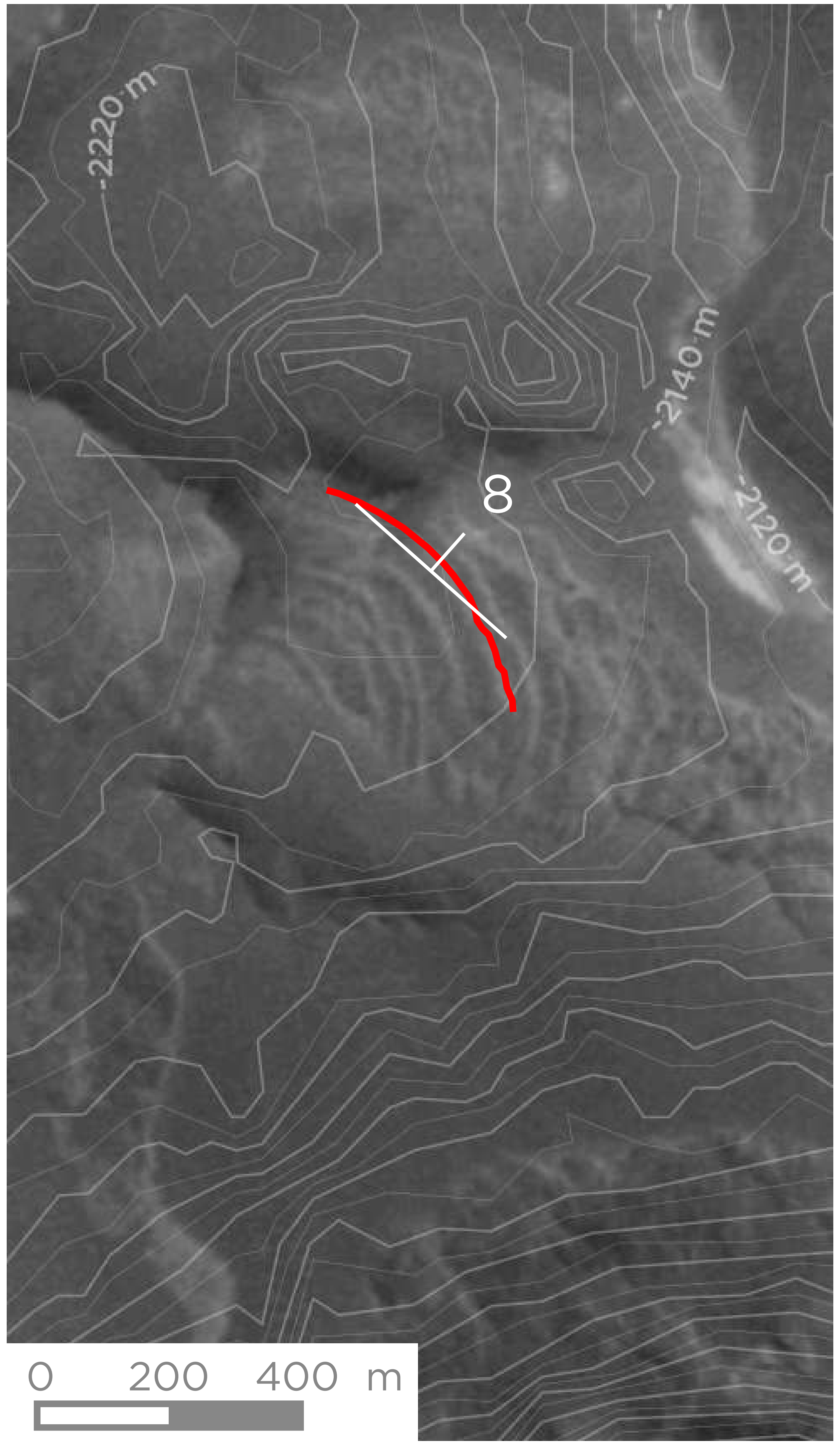
Orientation error structure: complimentary views of error space

Low-resolution data

Larger orientation errors



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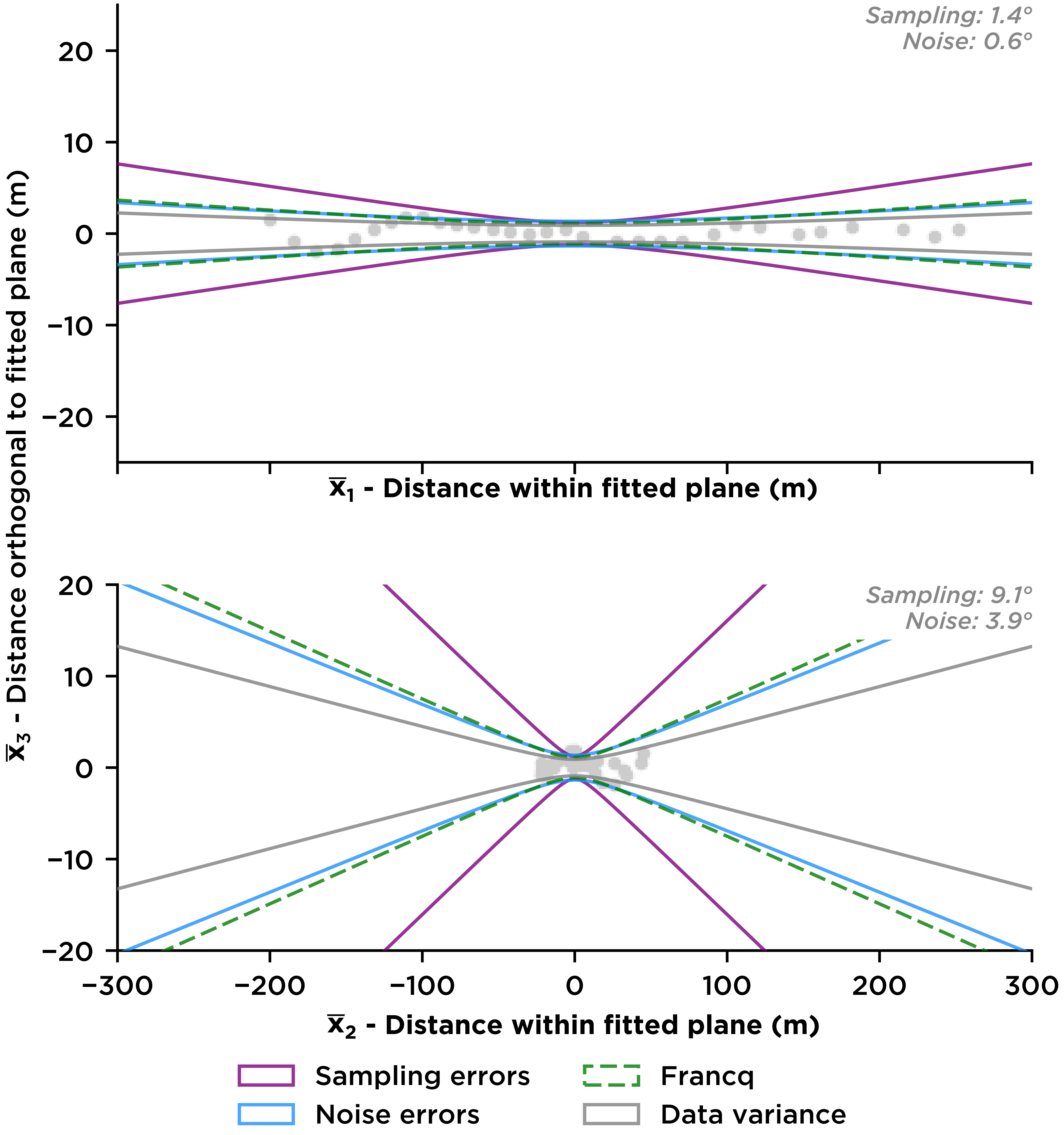

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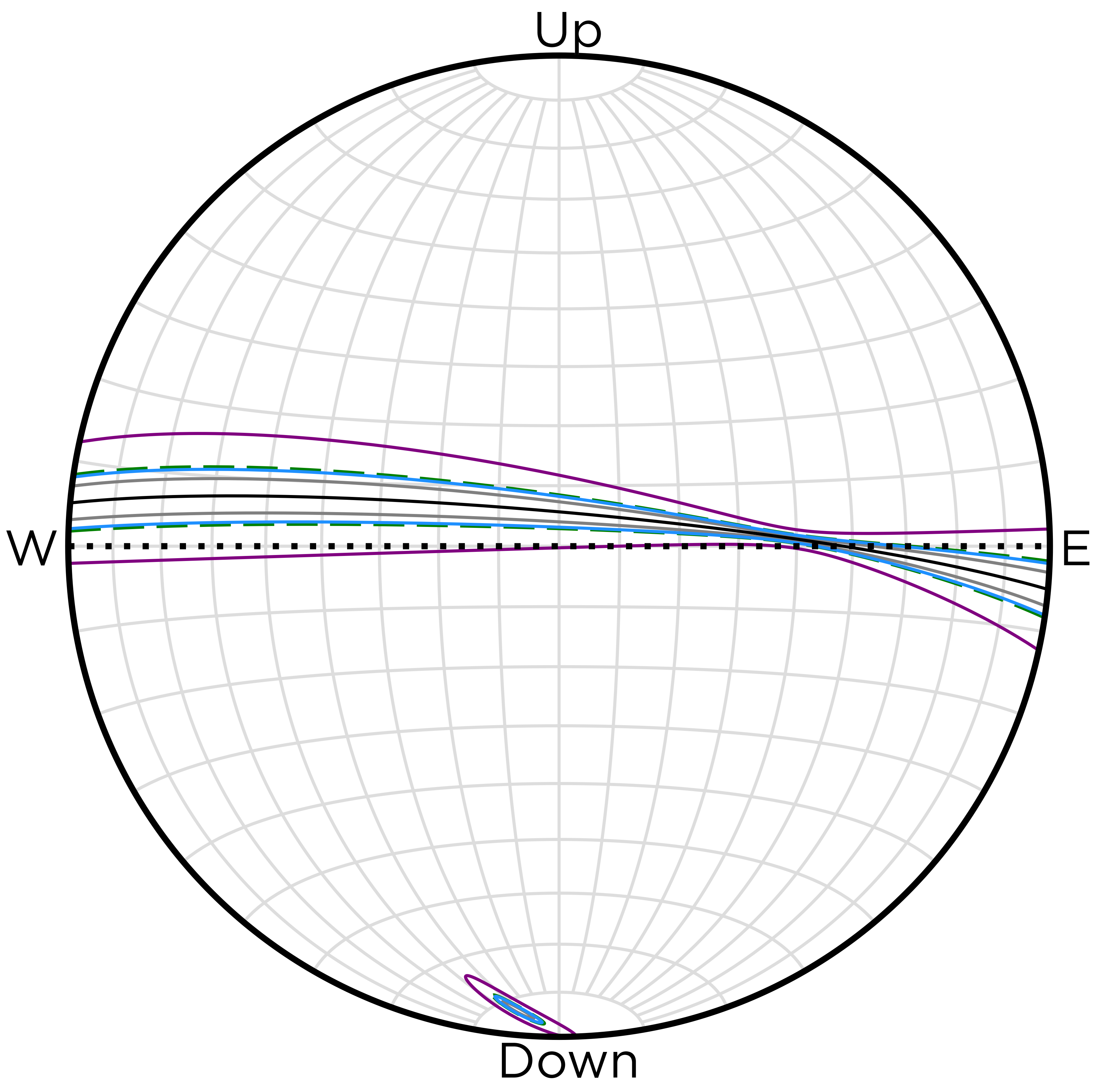

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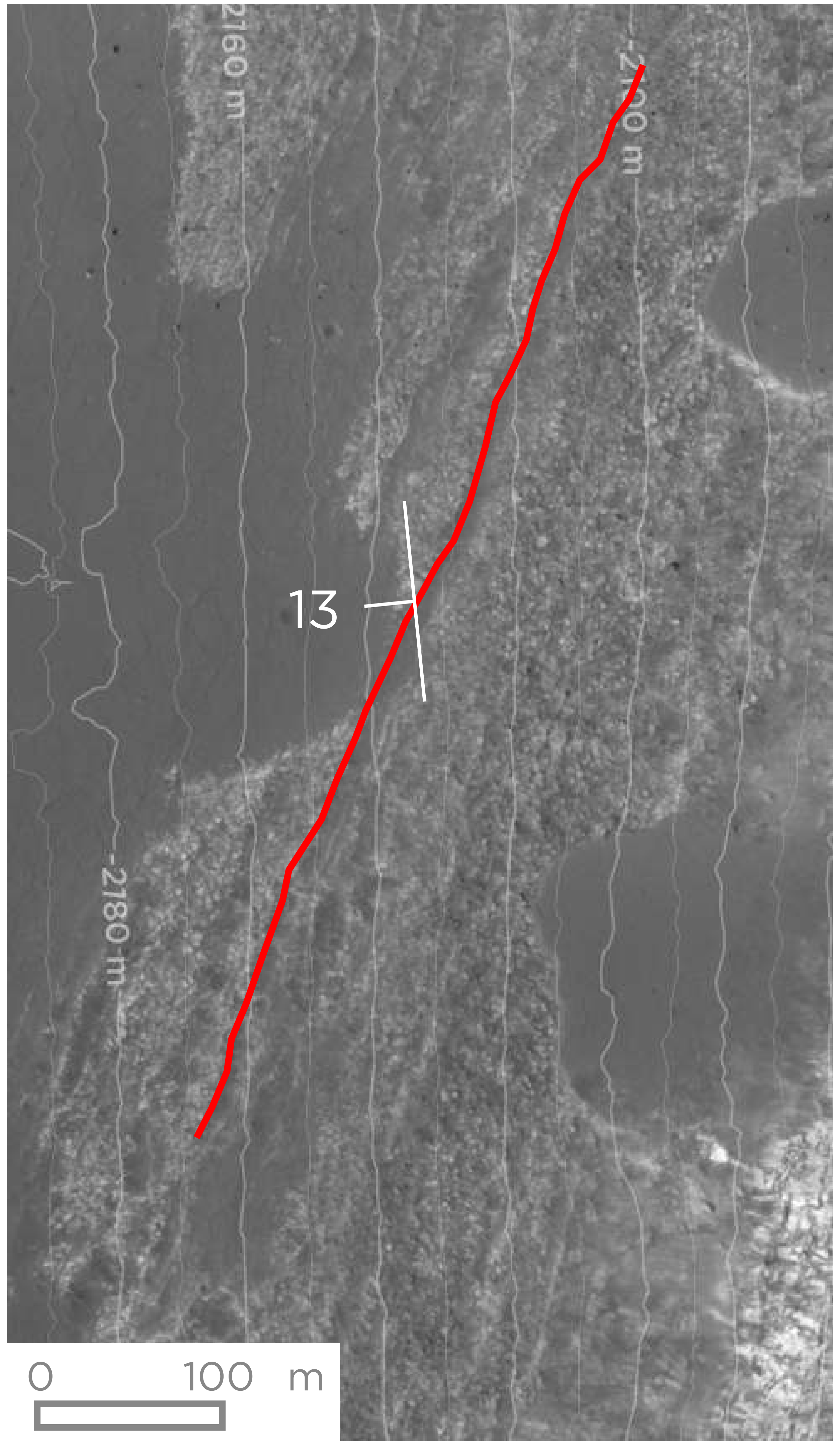

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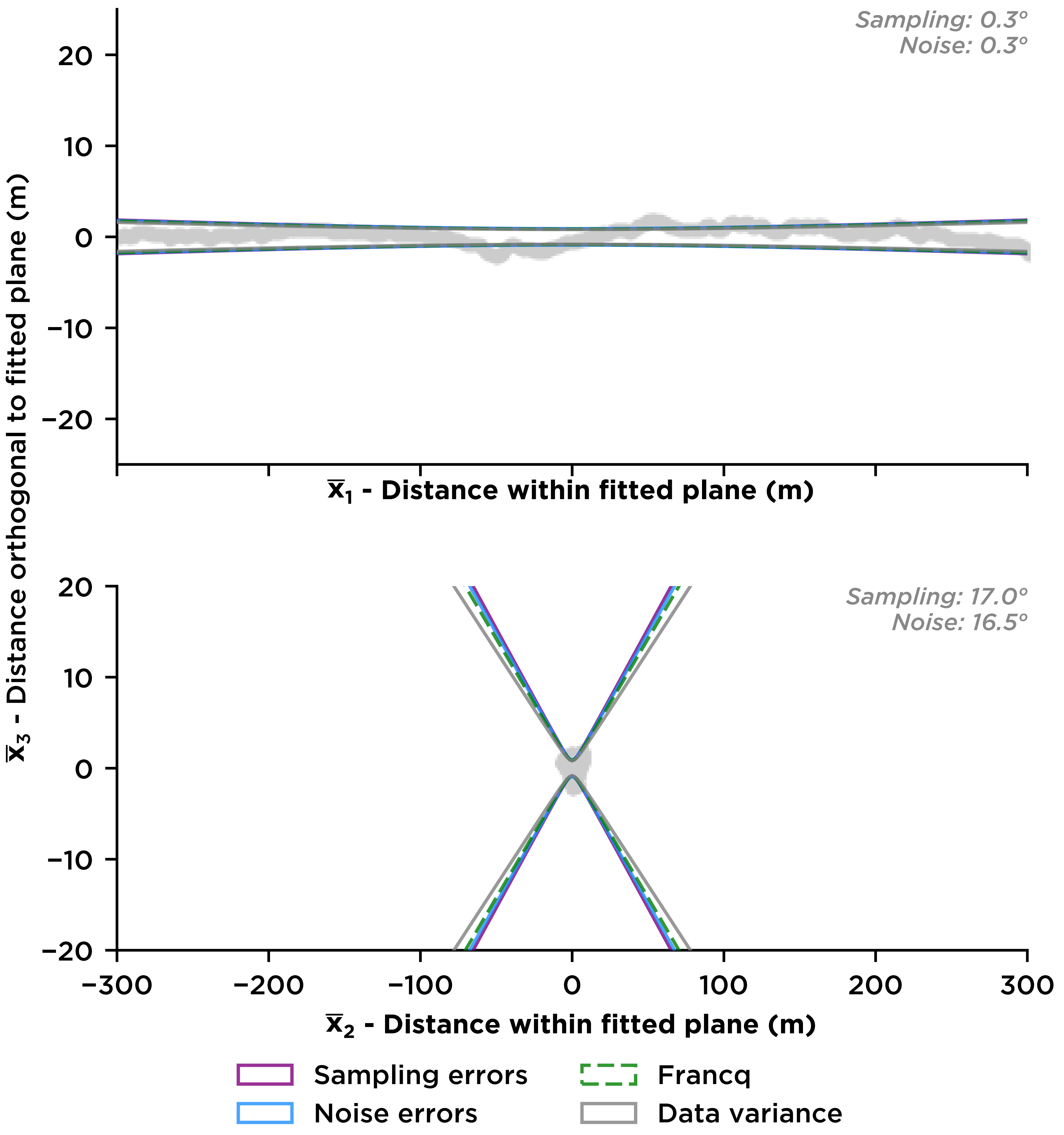

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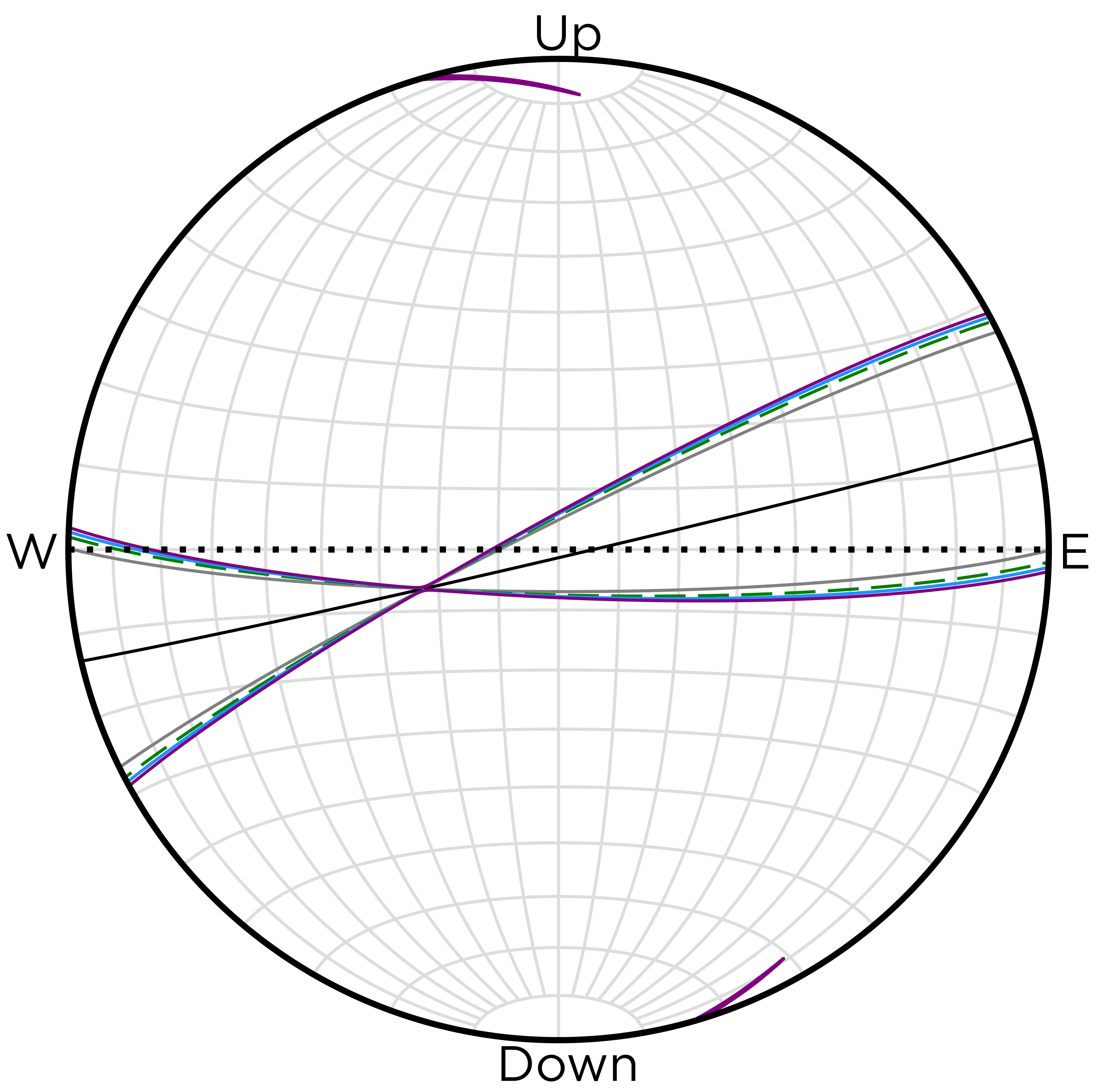

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Orientation error structure: complimentary views of error space

Poor-quality DEM

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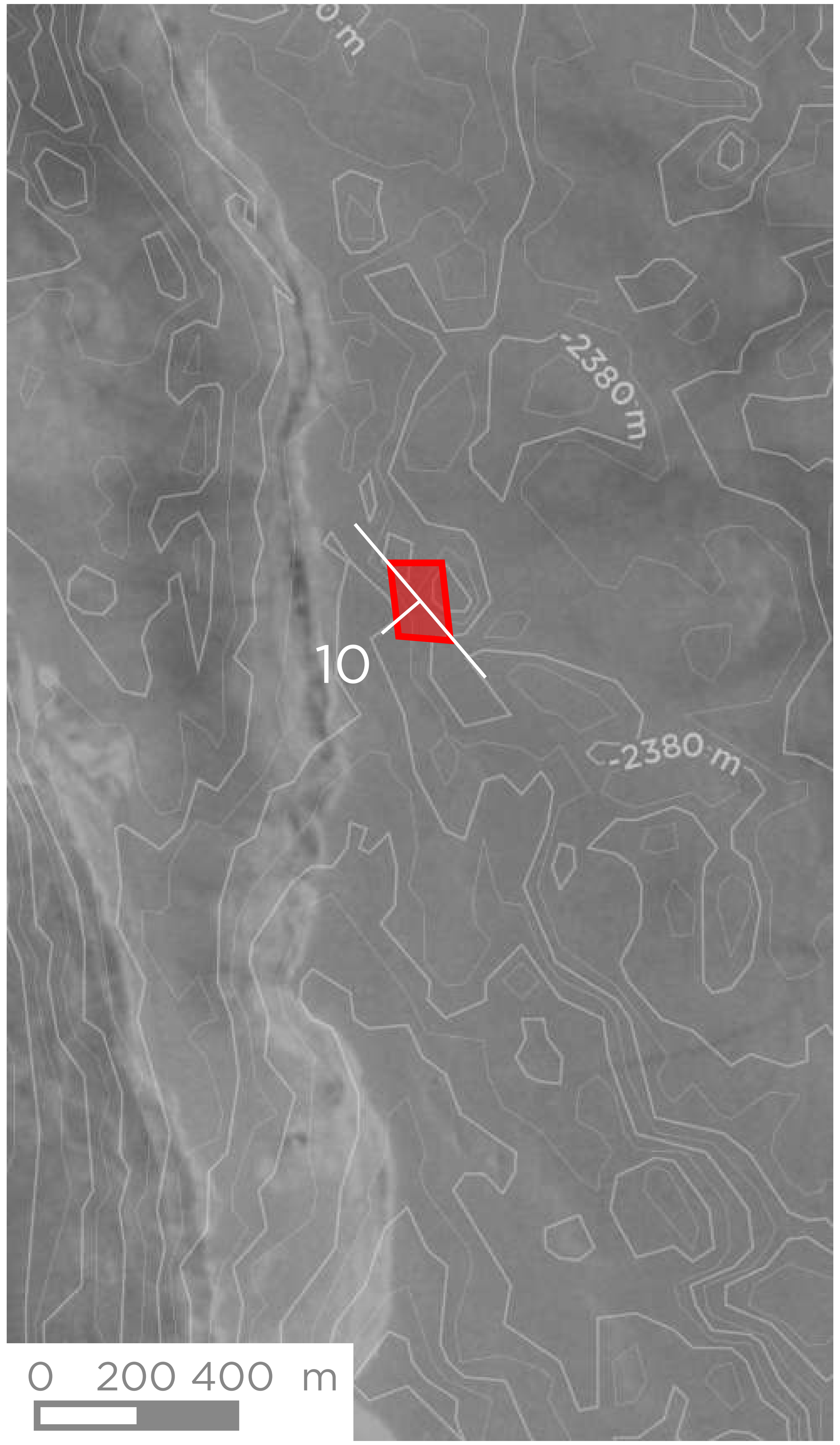

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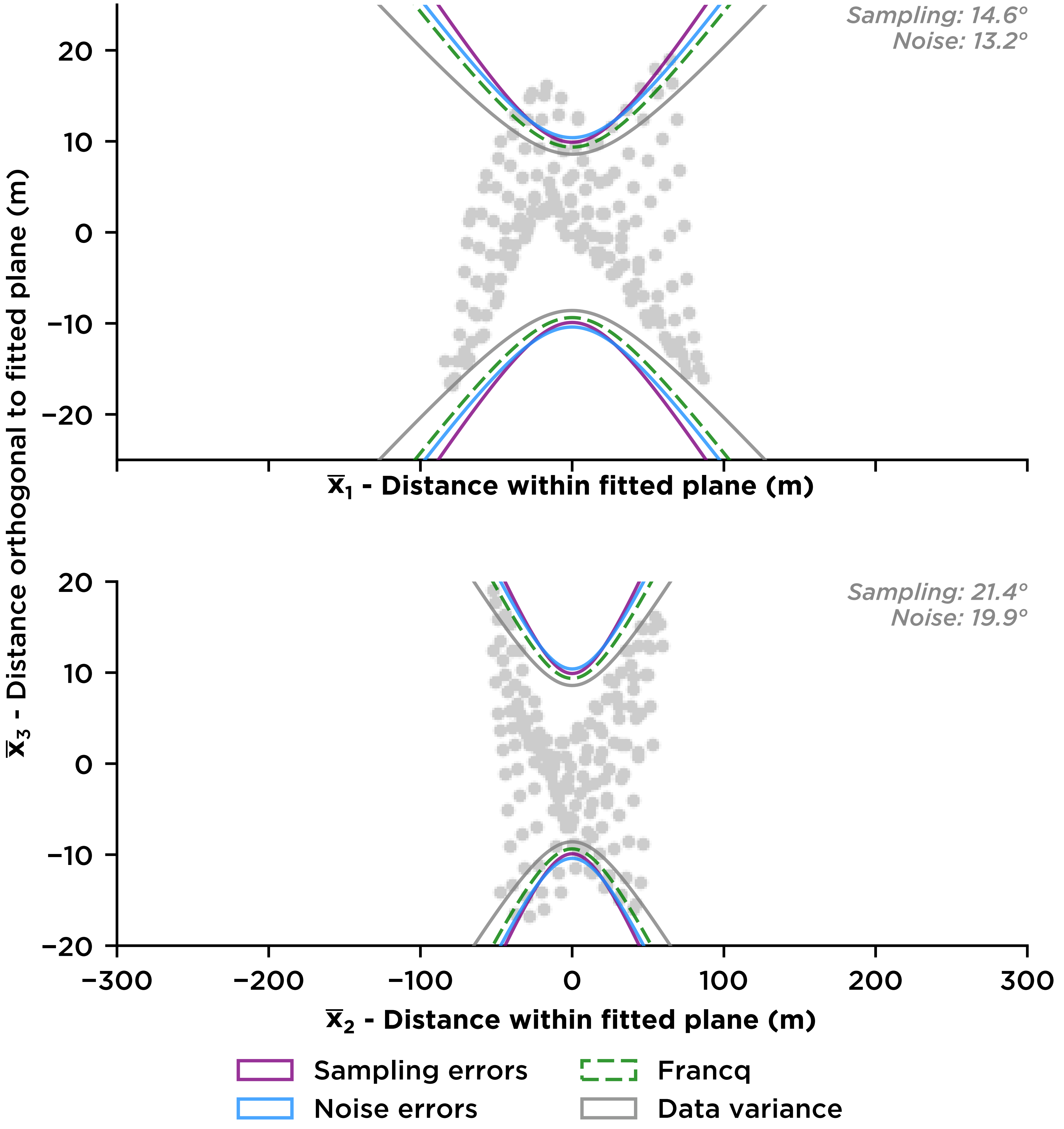

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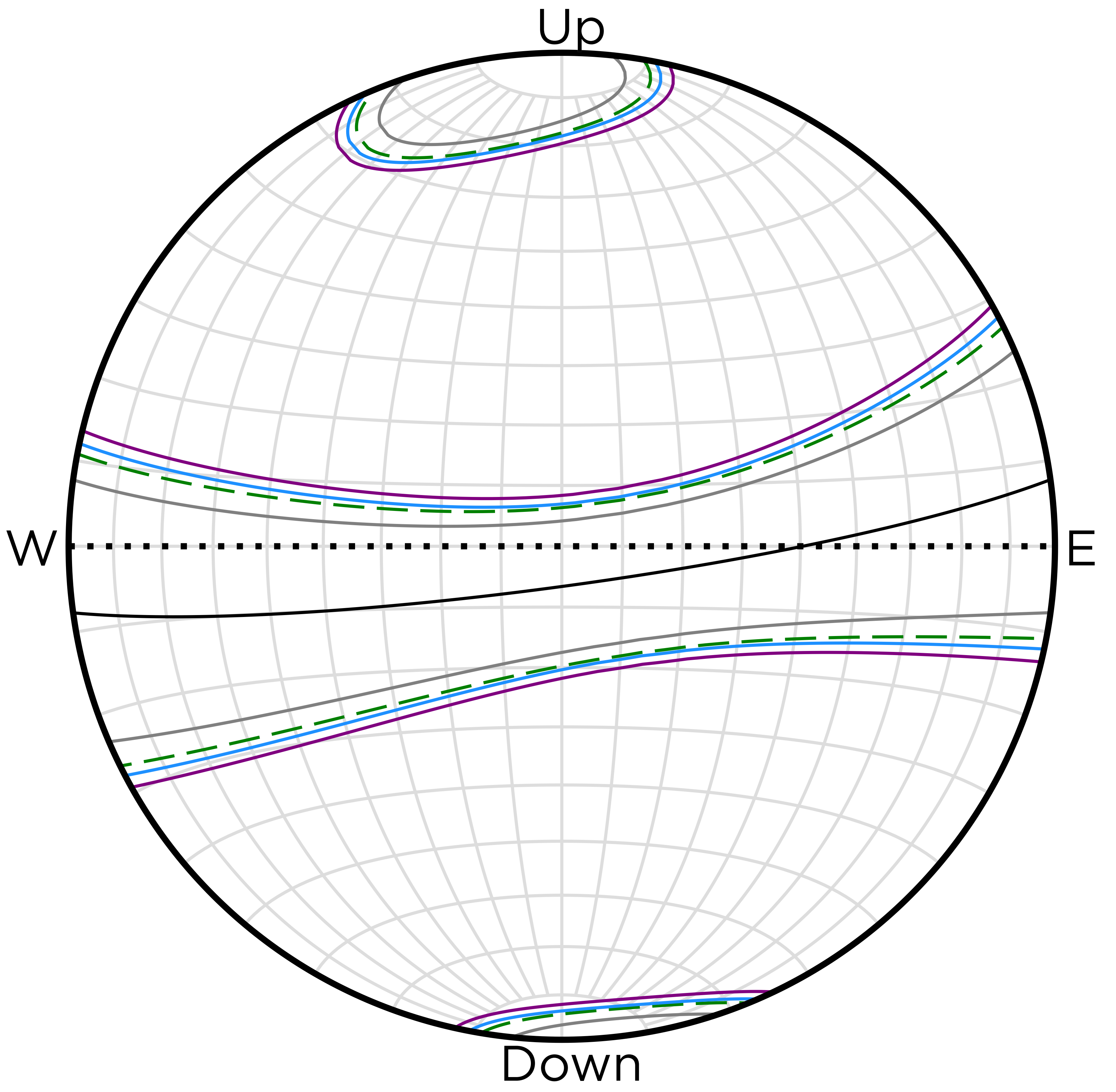

Upper-hemisphere, equal-area rotated stereonet

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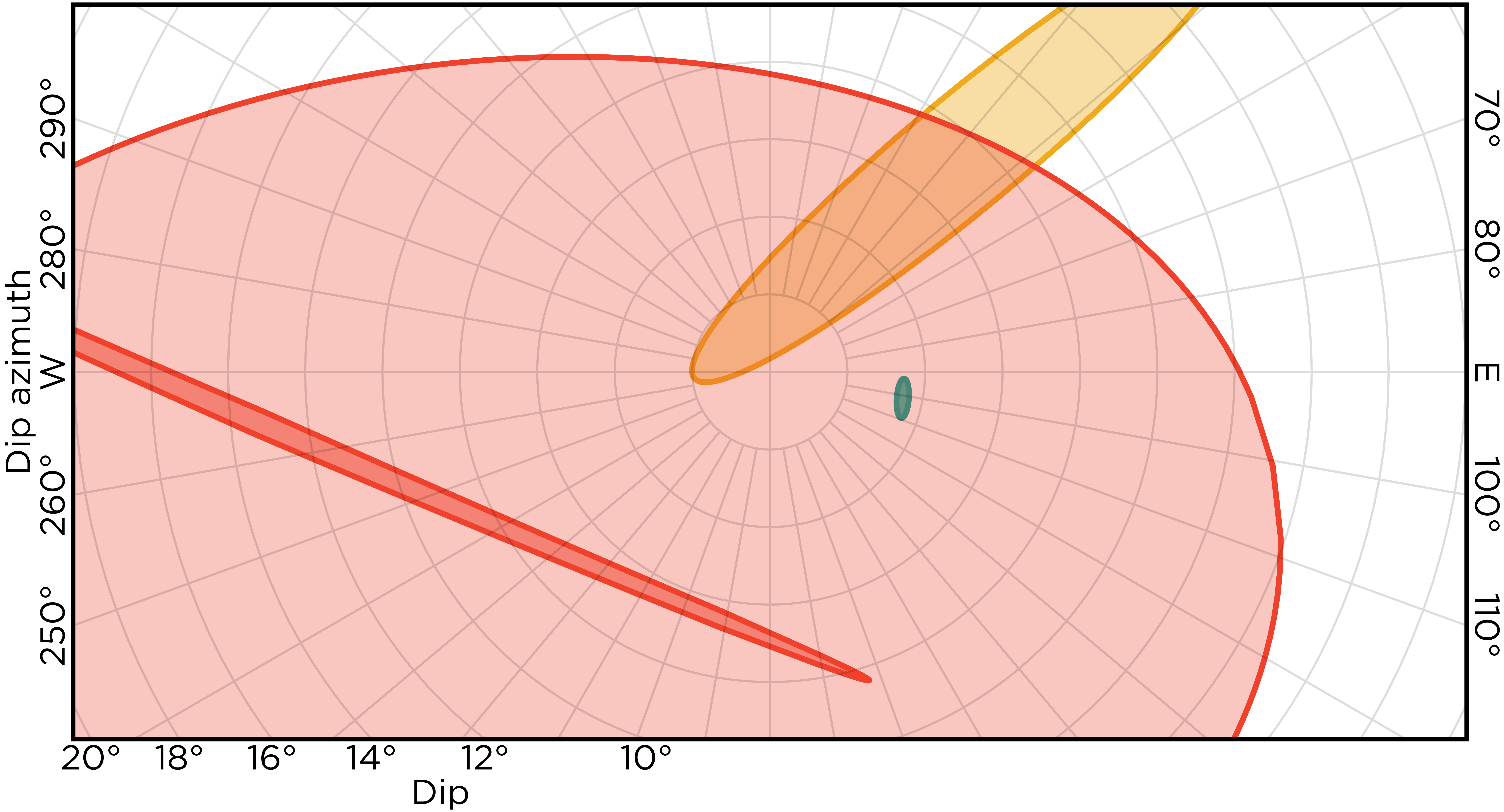
Orientation error structure: complimentary views of error space

Spherical dip/dip-azimuth space

- Shows entire error space
- Easily comparable for low-angle changes

Scale of angular errors

- Varies by orders of magnitude
- Highly nonlinear



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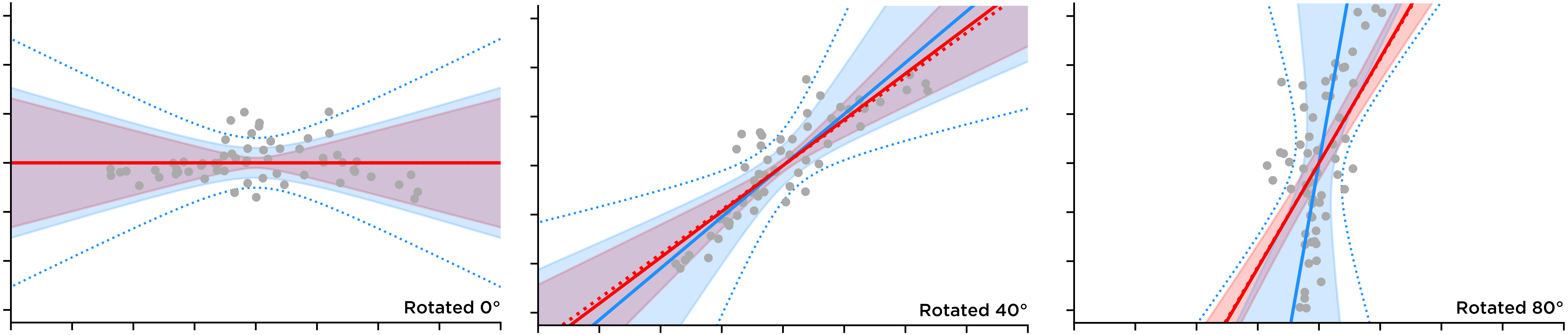
Adjustments to standard regression statistics

1. Principal component analysis (**PCA***) instead of ordinary least squares (**OLS**)

- **In OLS, errors are always vertical** (implicit test against horizontality)
- **PCA is rotation-invariant** (scale of errors does not depend on the orientation of the point cloud)
- **Important for comparison between different hillslope orientations and view geometries**



Same test point cloud, rotated by 0, 40, and 80°: PCA fit captures rotation while OLS is biased towards shallower orientations; PCA fit also retains the same error structure.

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****PCA** as used here is functionally equivalent to total least squares (**TLS**)*

Adjustments to standard regression statistics

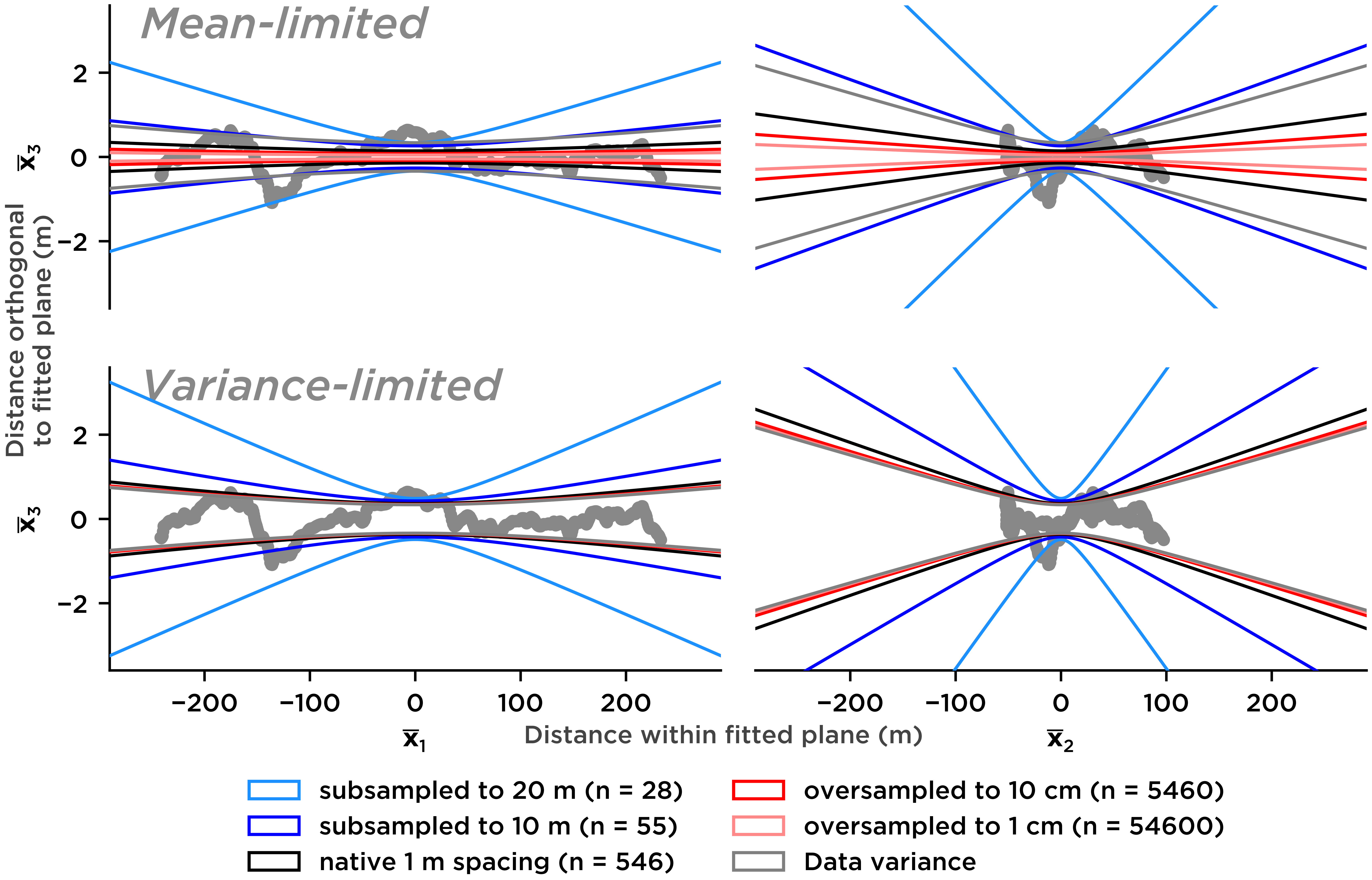
2. Parametrization of central limit: variance of data instead of mean

Mean-limited regression

- Errors decrease to zero when oversampled
- Insensitive to scattered data
- Regression line/plane represents the center of a scattered point cloud

Variance-limited regression

- Errors decrease to variance when oversampled
- Much more sensitive to highly scattered planes



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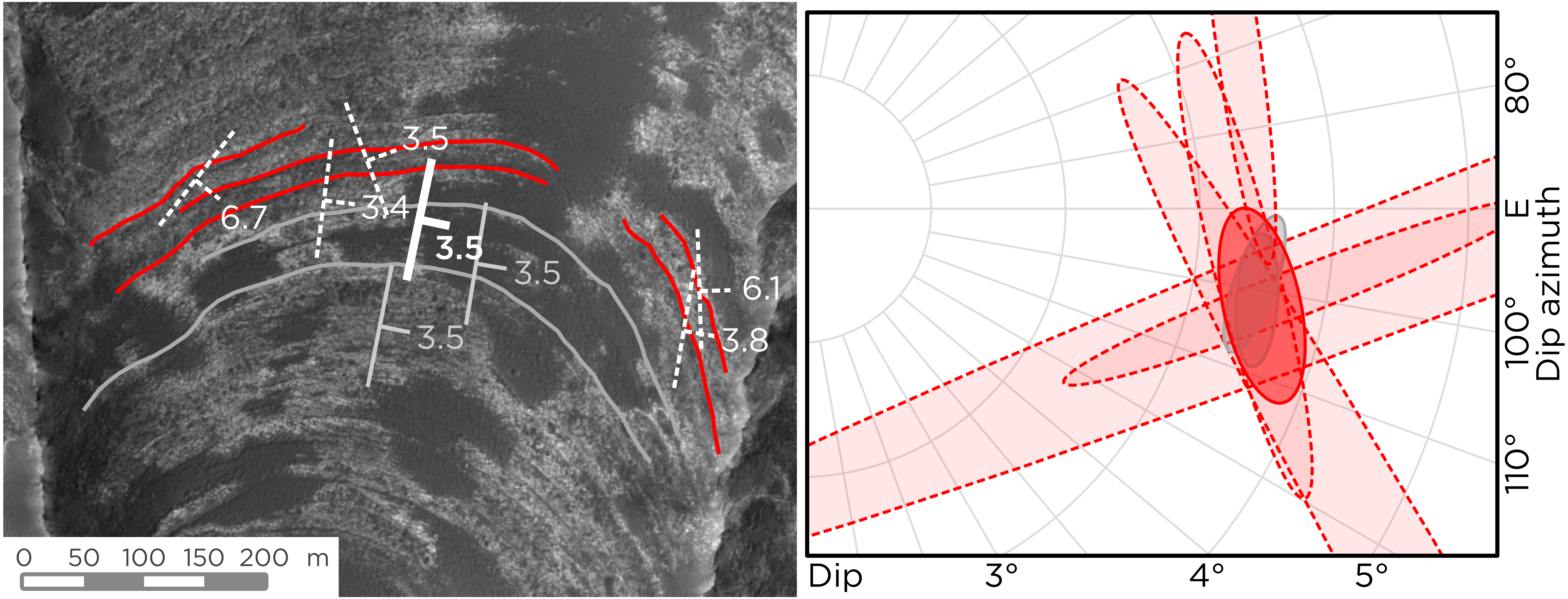
Adjustments to standard regression statistics

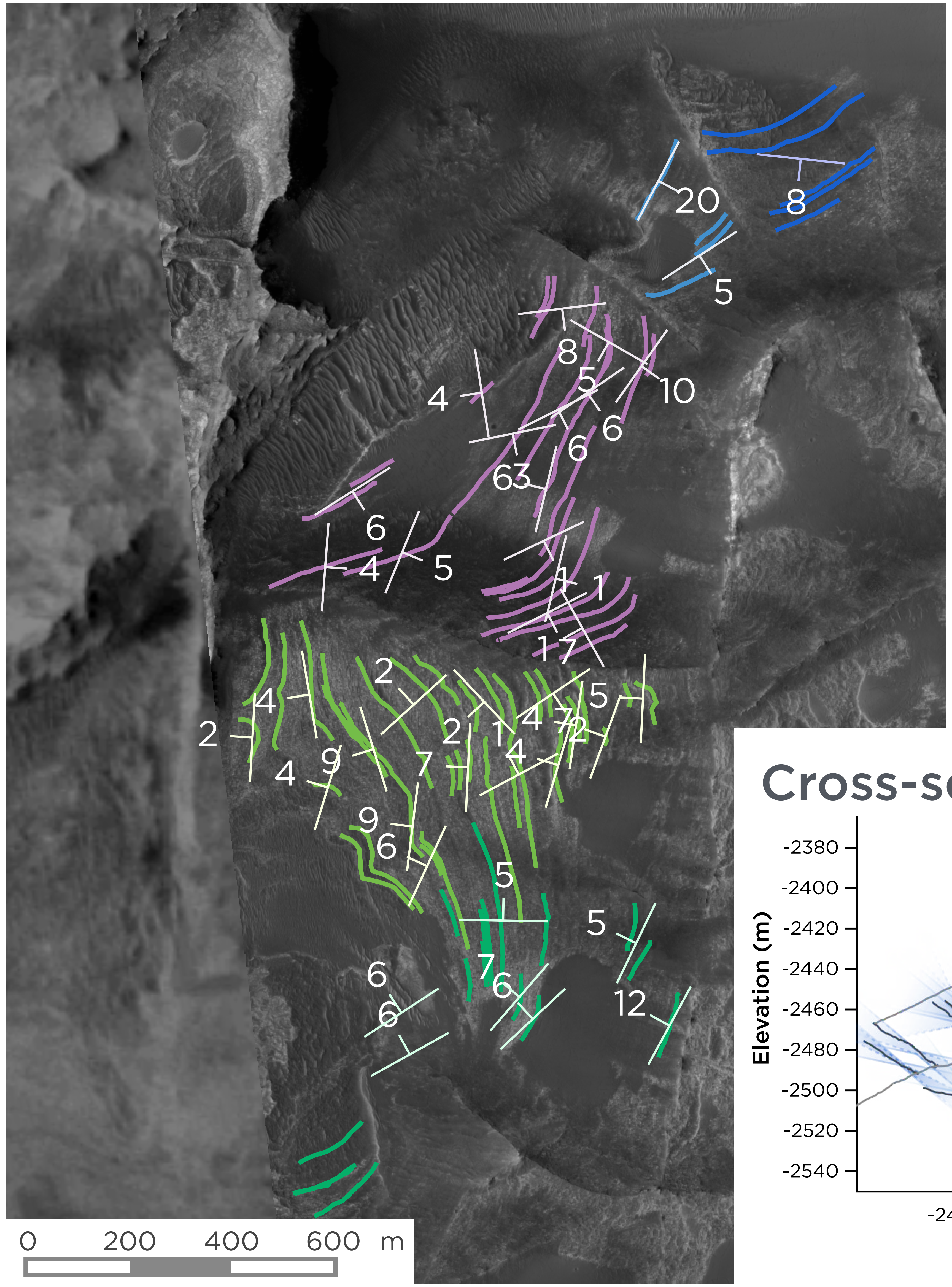
3. Joint fitting of potentially parallel planes

Use assumption of parallel bedding to "stack" and jointly fit adjacent beds with different error structures

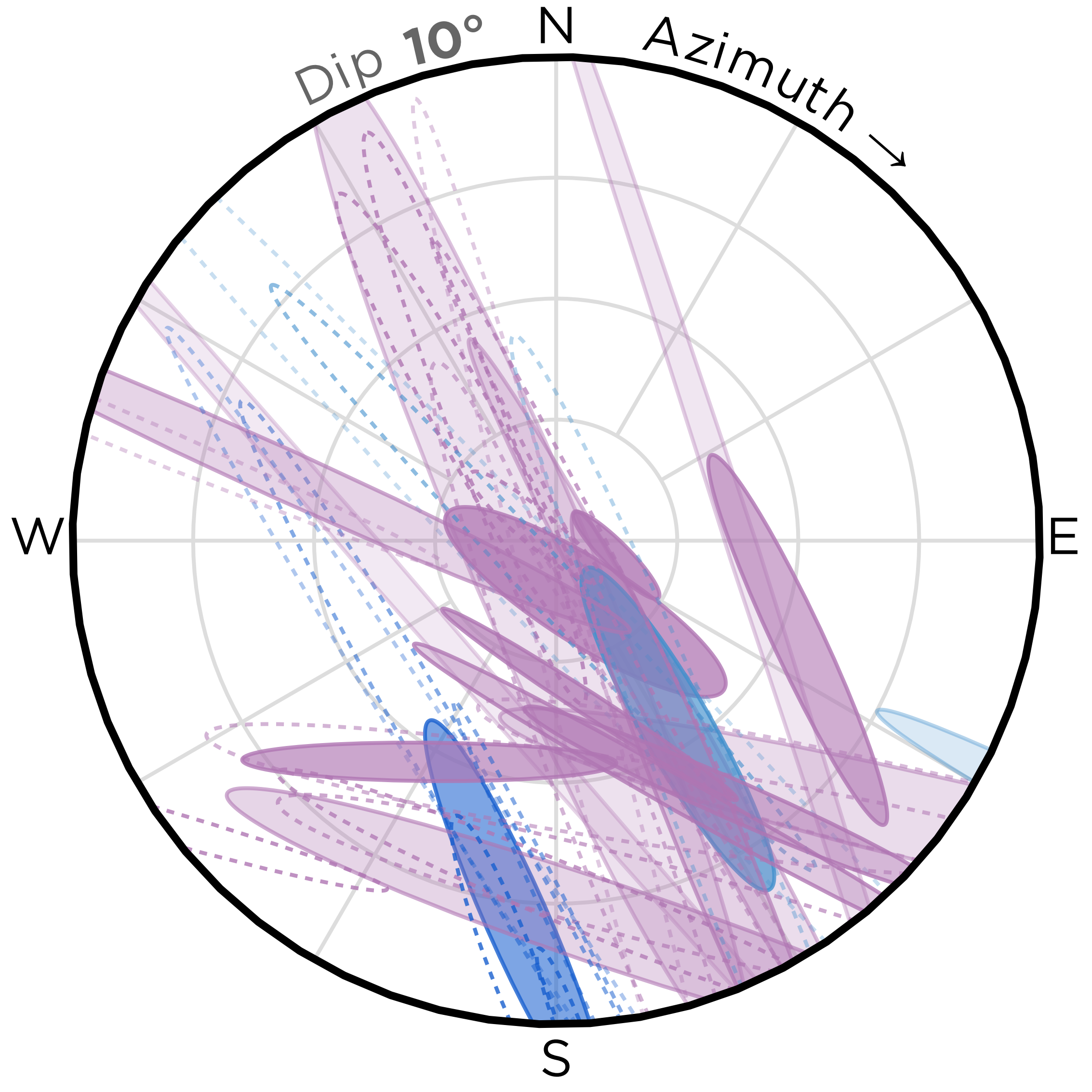


Applications: the motivating project
Northeast Syrtis, Mars — localized structure

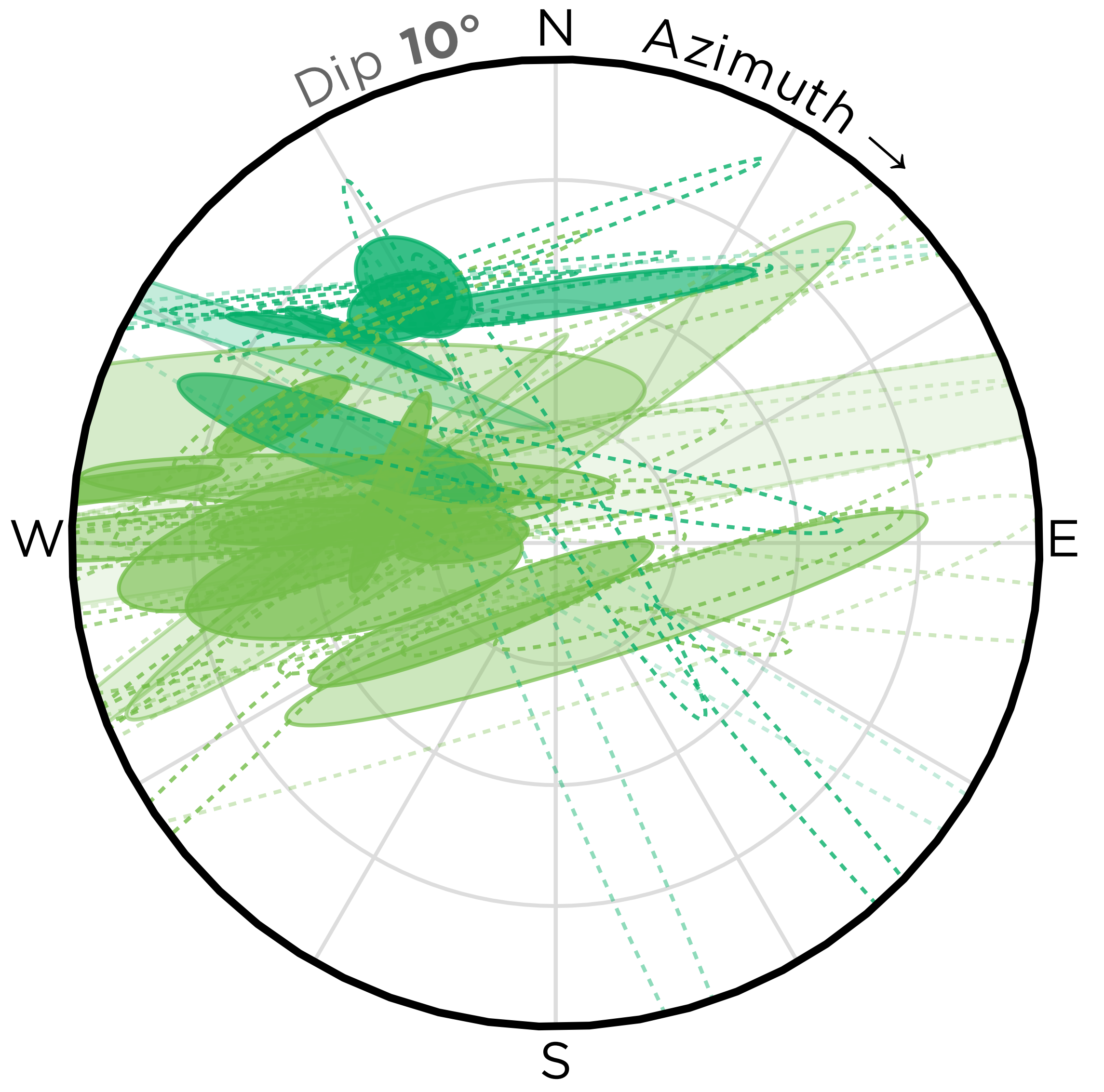
Clear change in dip orientation between boxwork fracture zones



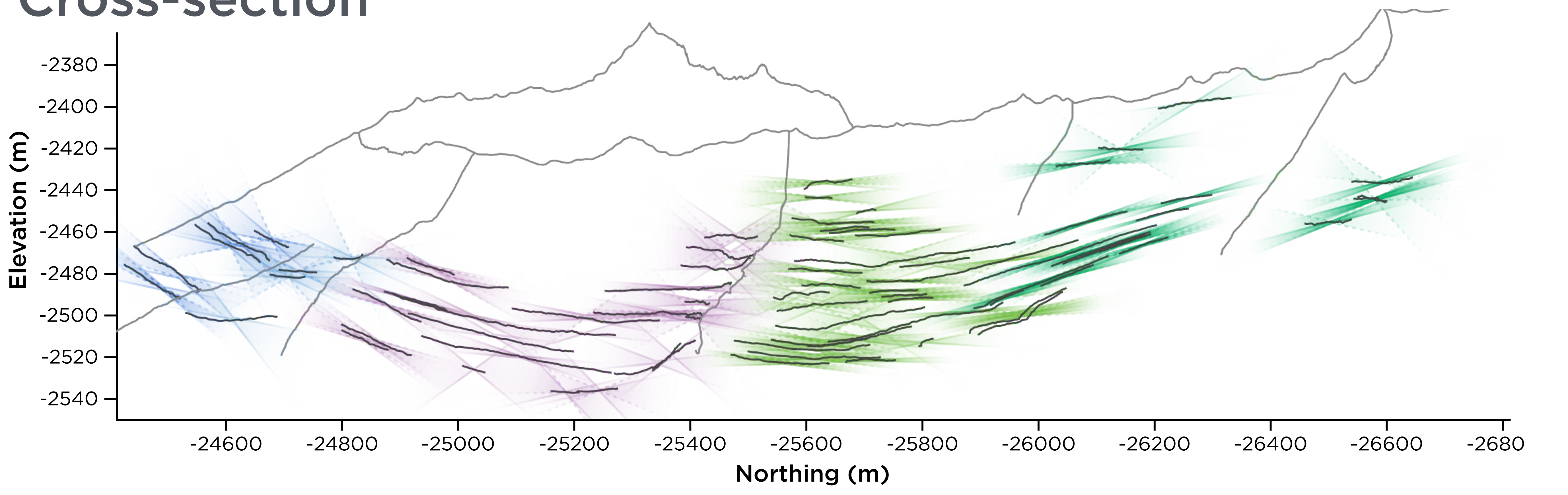
North orientations



South orientations

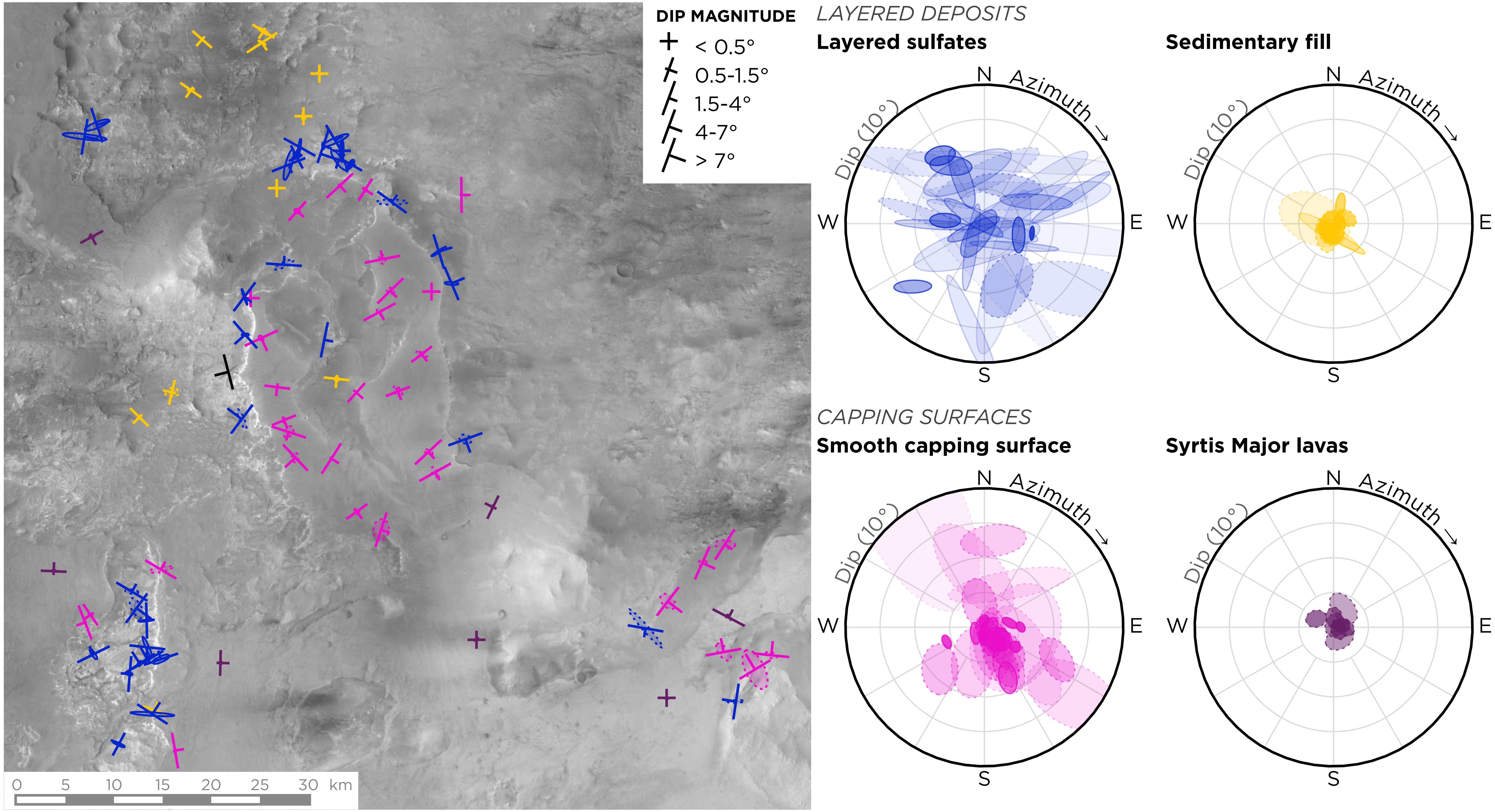


Cross-section



Applications: the motivating project
Northeast Syrtis, Mars — regional mapping

- Regional summary of bedding orientations
- Comparison across different datasets and outcrop types
- **Conclusion**: the layered sulfates show features of draping sedimentary deposits



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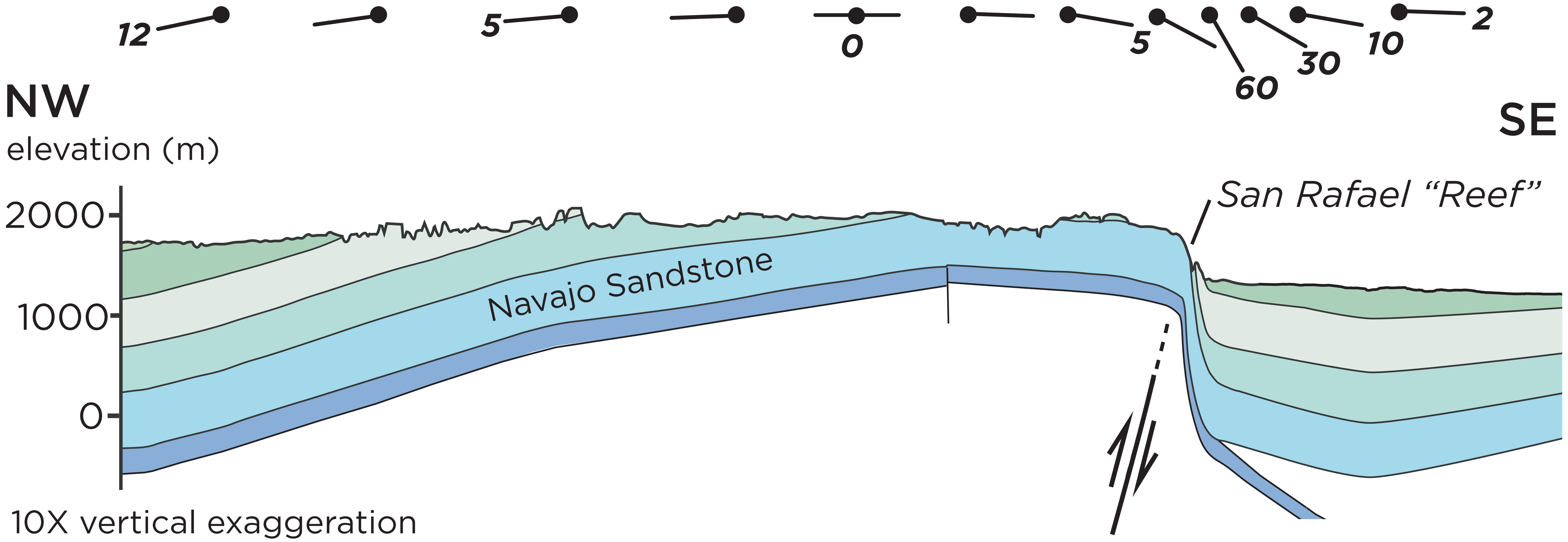
Applications: aerial DEM and satellite imagery of Earth
San Rafael Swell, south-central Utah, USA

A world-class structural test case



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Applications: aerial DEM and satellite imagery of Earth
San Rafael Swell, south-central Utah, USA

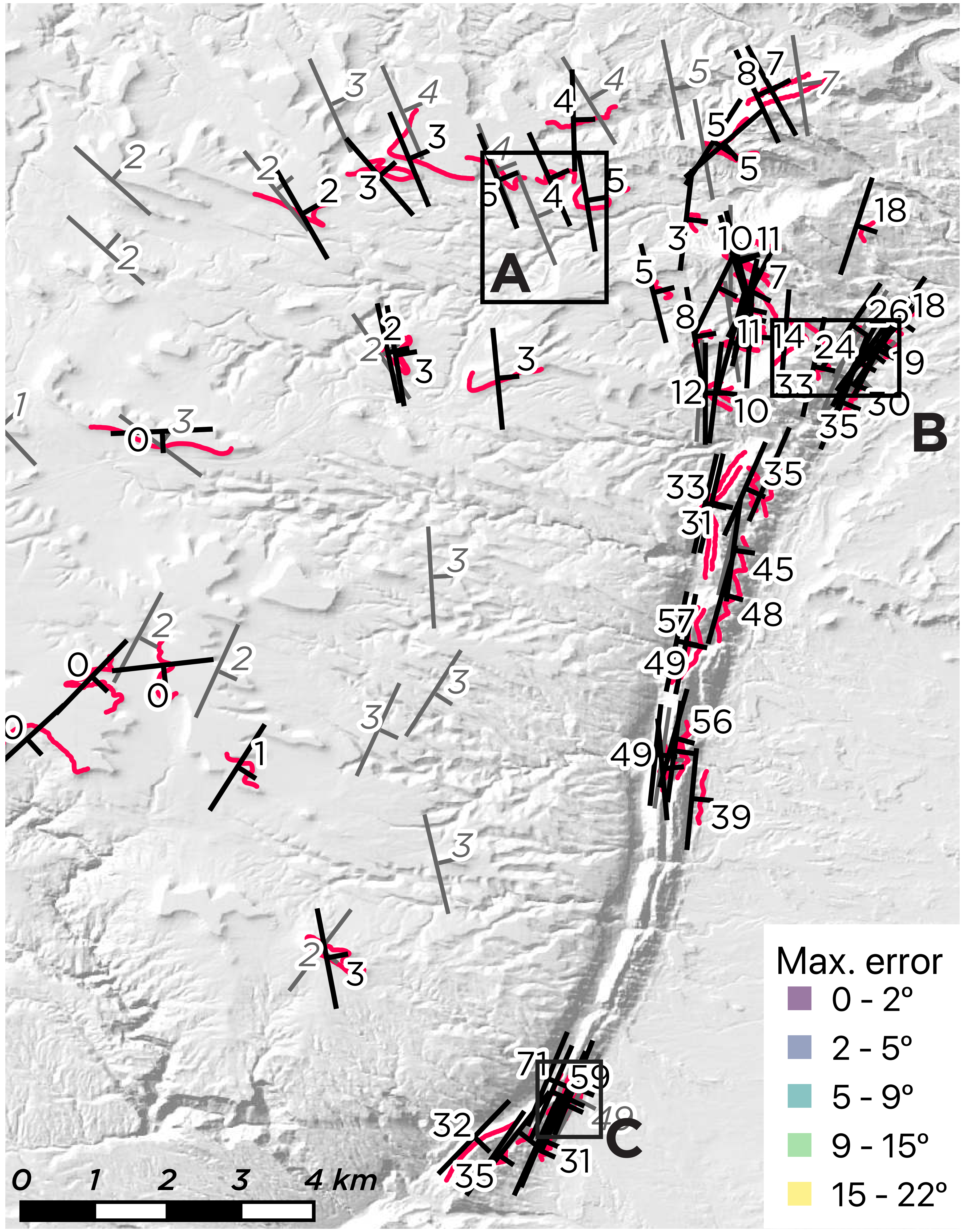
Used readily available data

- Google Maps orthoimages
- State-provided aerial DEM
- No alignment or post-processing

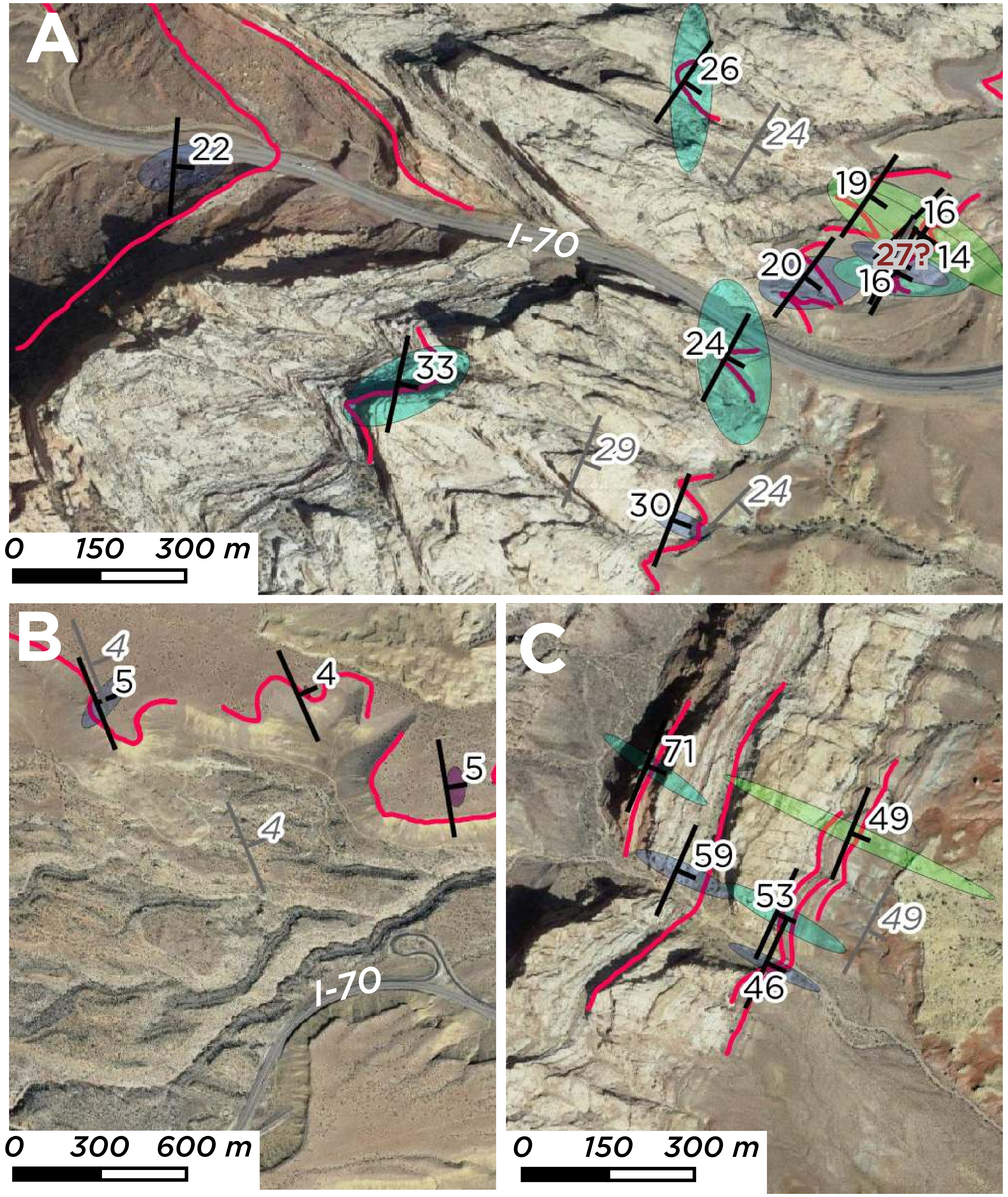
Results

- Accepted only planes with low errors and residuals
- >60 planes had high-quality fits
- Replicated both broad structural pattern and individual measurements

5-m/px aerial photogrammetric DEM



Google Maps orthoimagery (~1 m/px)



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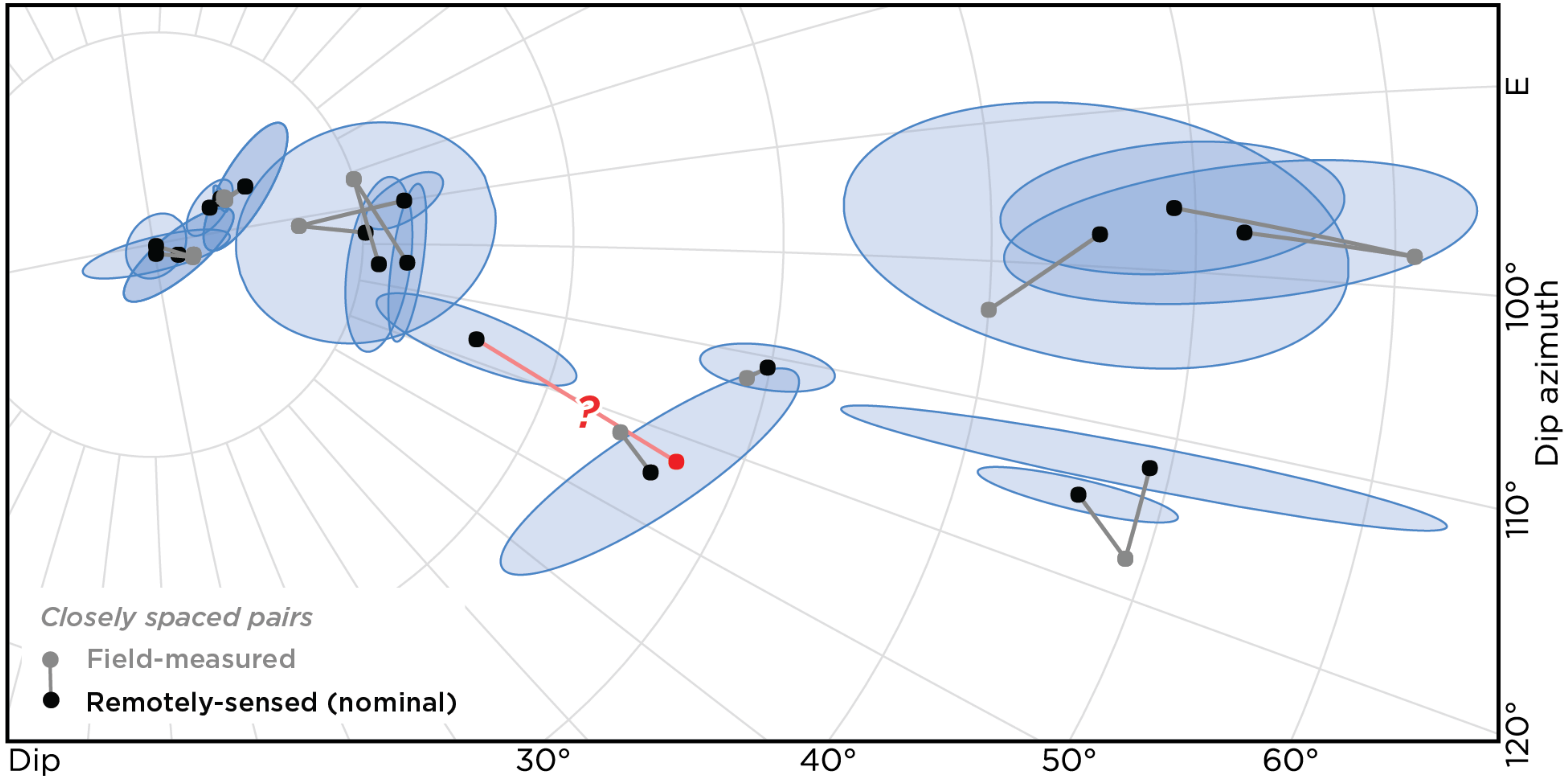
Remotely-sensed orientations generally correspond within error to nearest literature measurement



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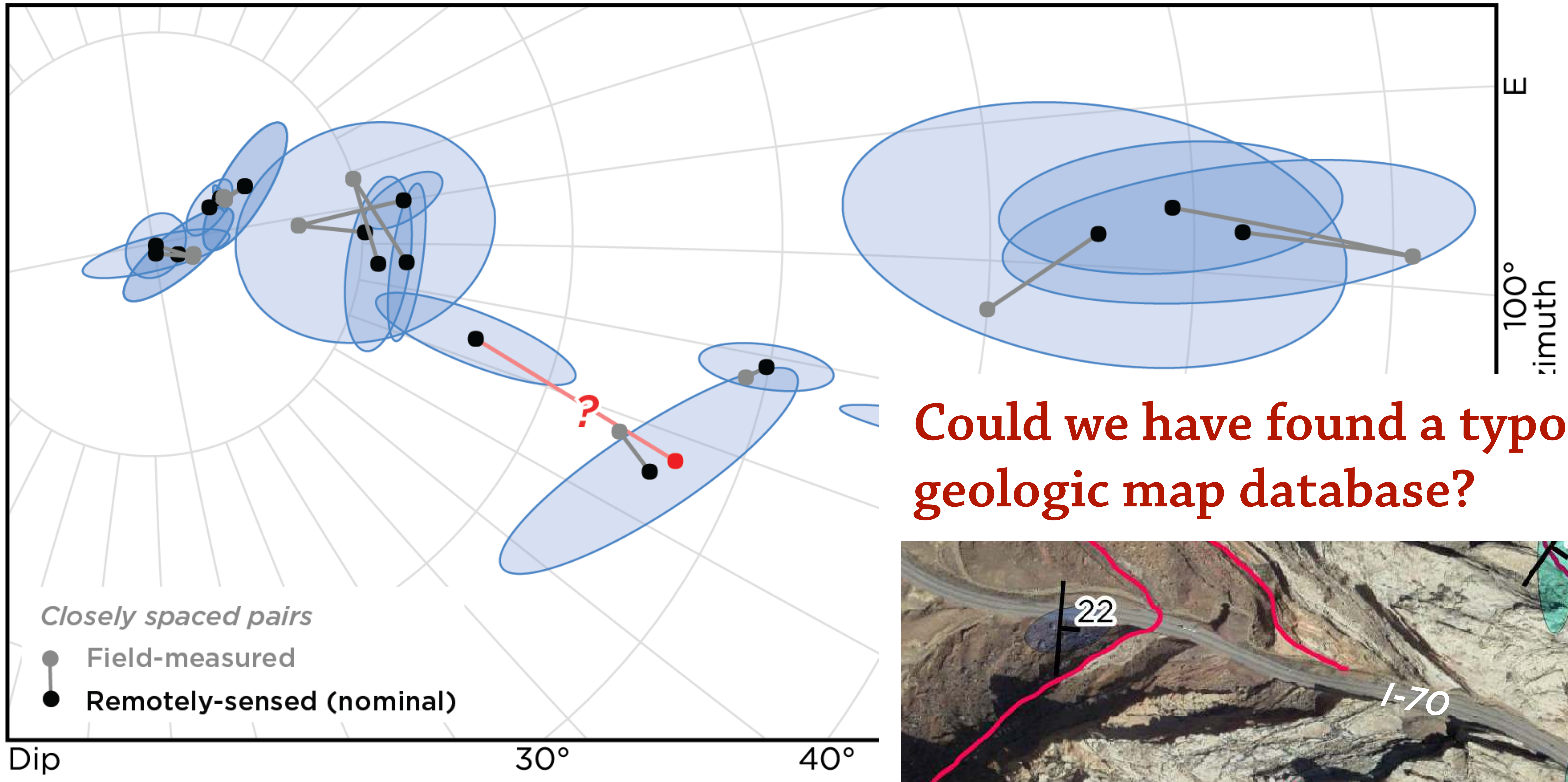
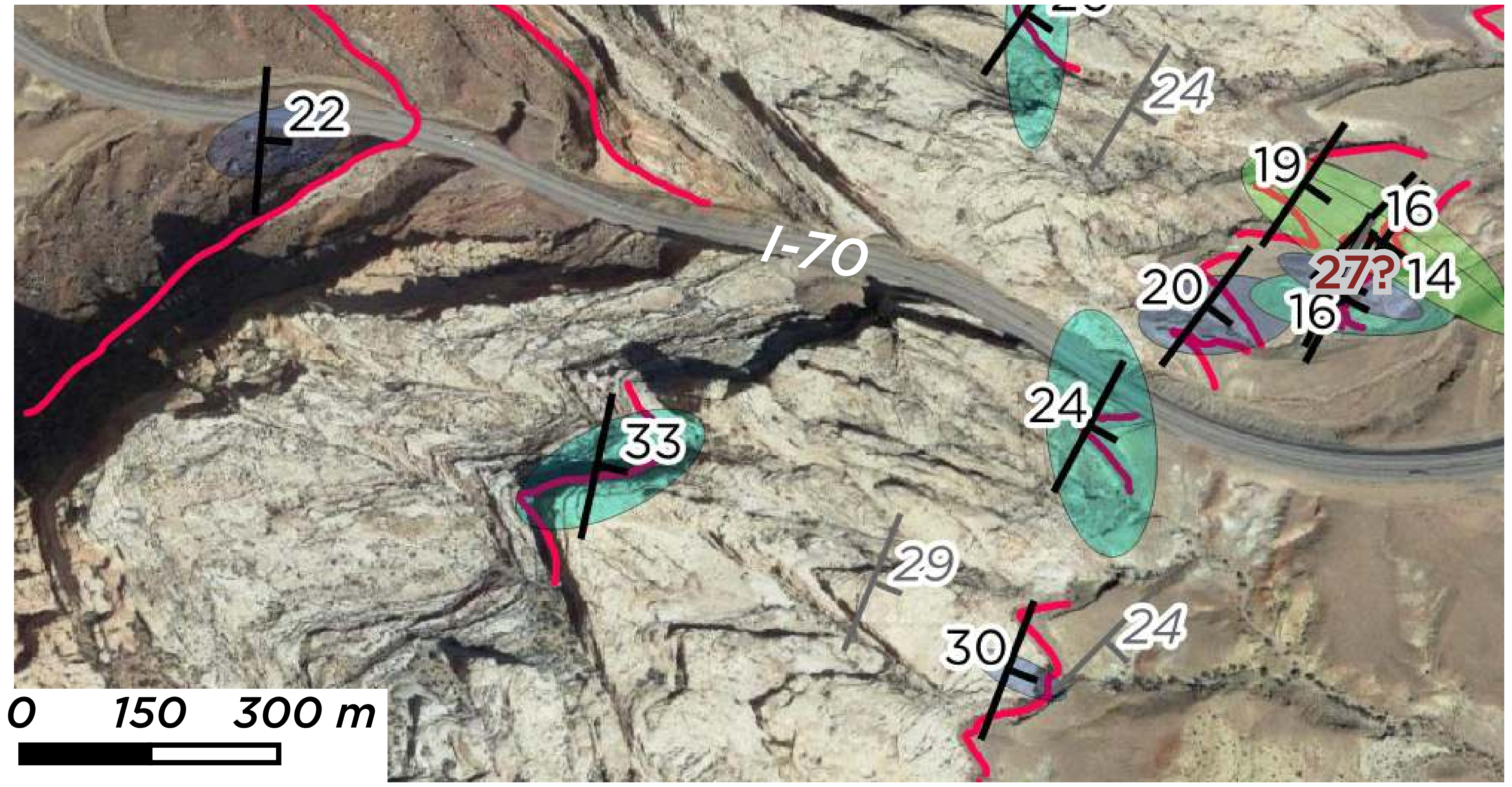


Could we have found a typo in the geologic map database?



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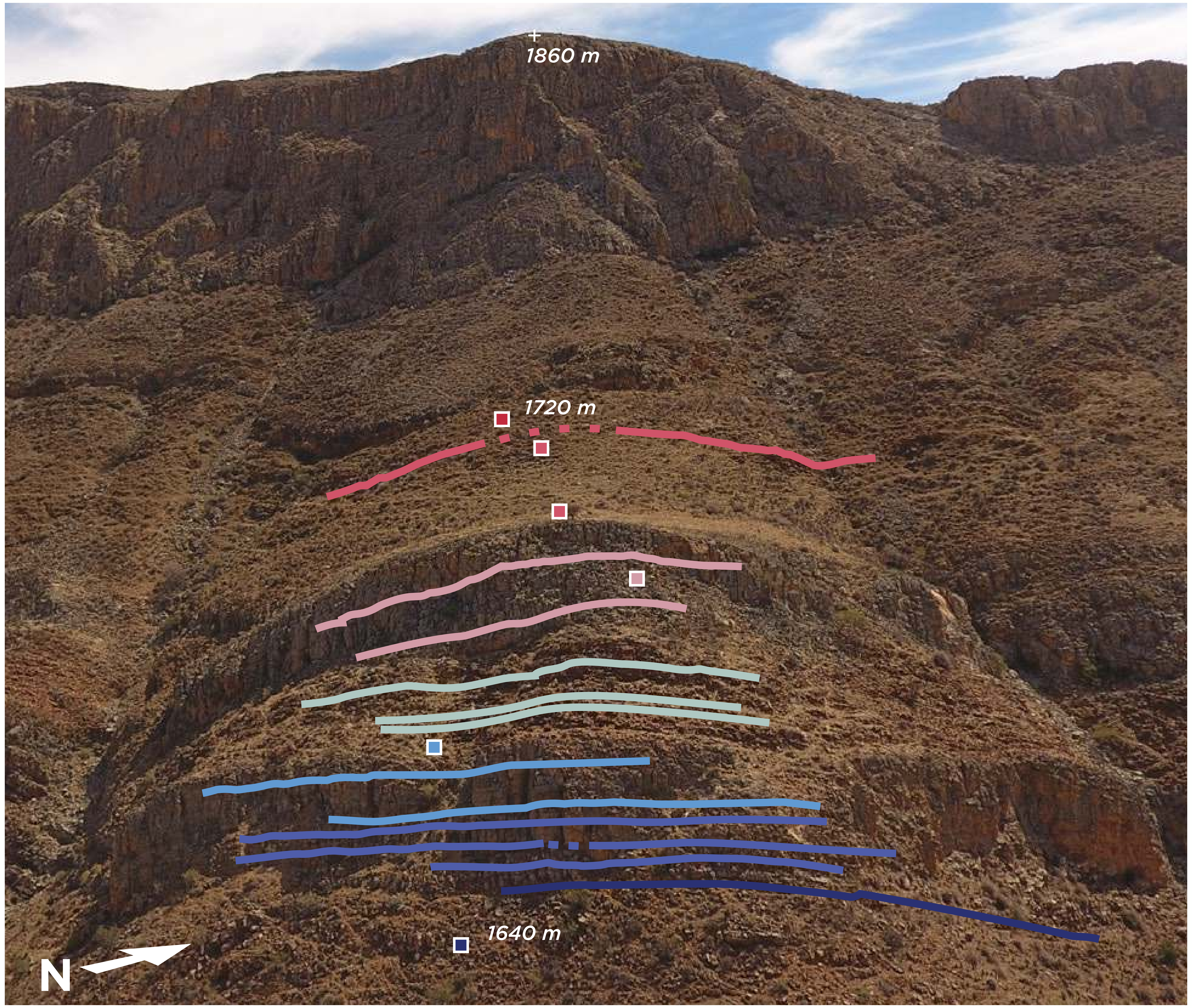
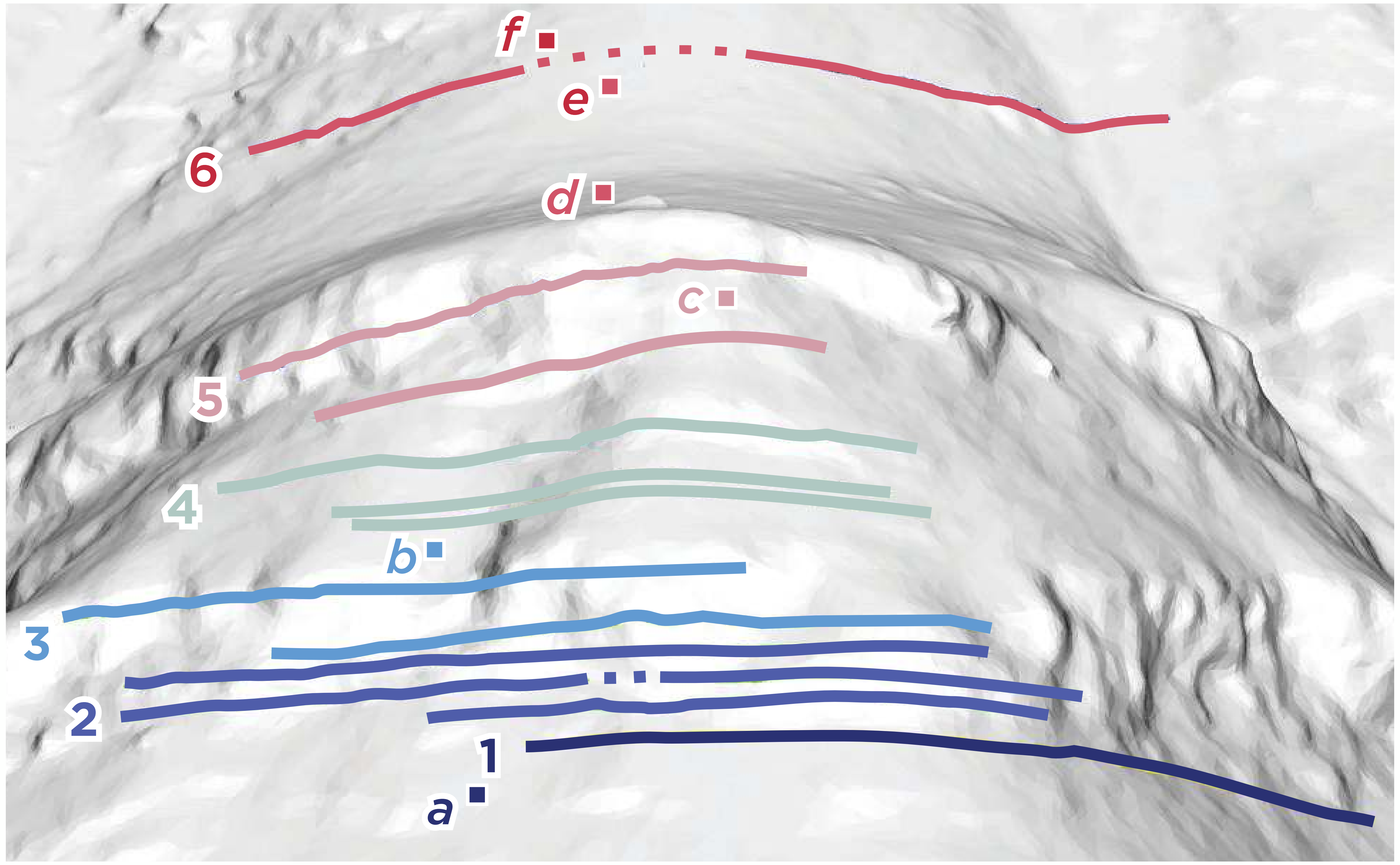
Applications: UAV structure-from-motion models
Onis cliffs, Naukluft Mountains, Namibia



Methods

- DJI Phantom 4: one flight with 500 m standoff
- Low-resolution/quality 3D model in Agisoft Photoscan
- Extracted 3D linework in Agisoft Photoscan atop textured mesh and densified using Python script
- Direct measurement of six bedding orientations using a structural compass for comparison



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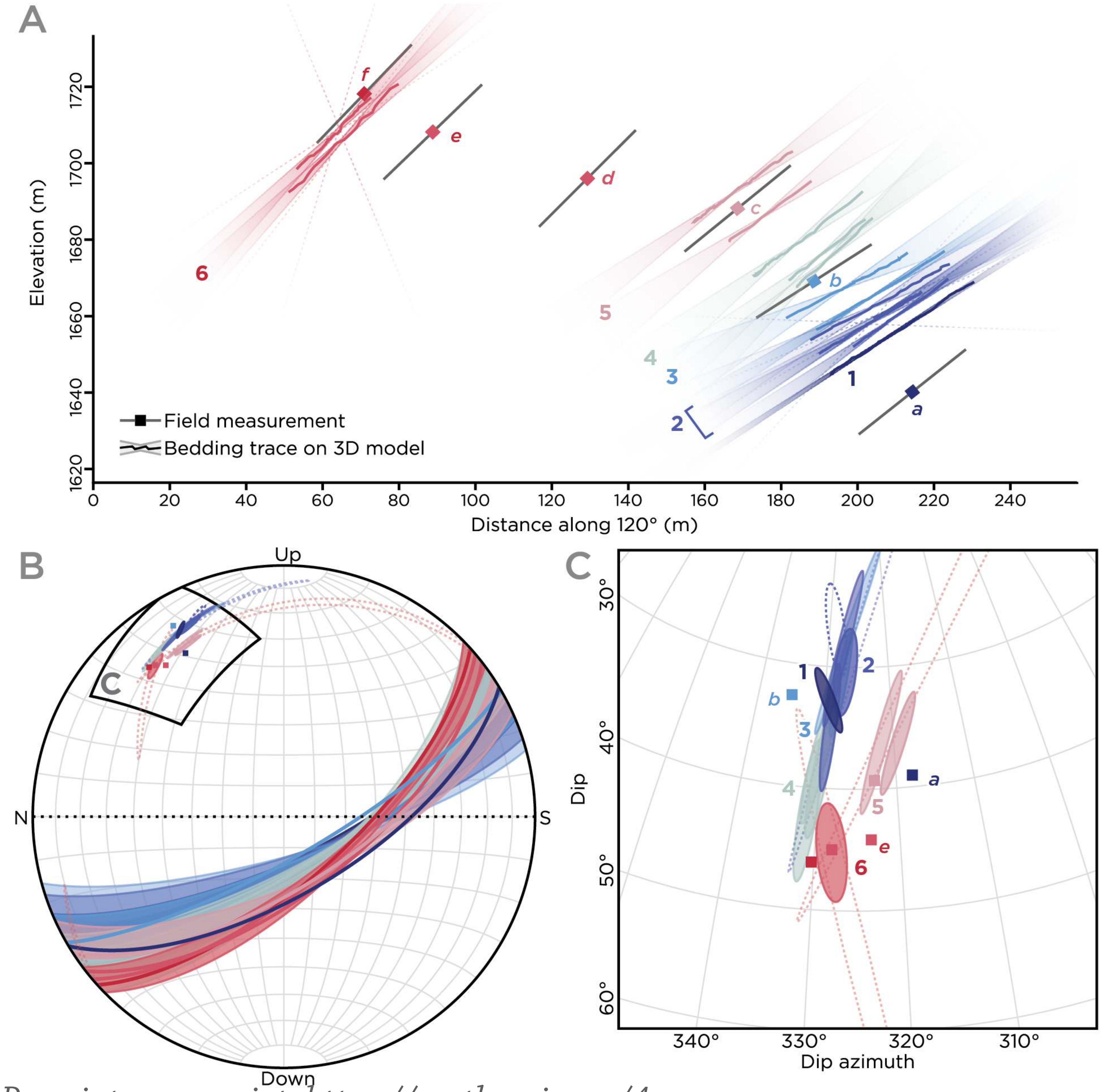
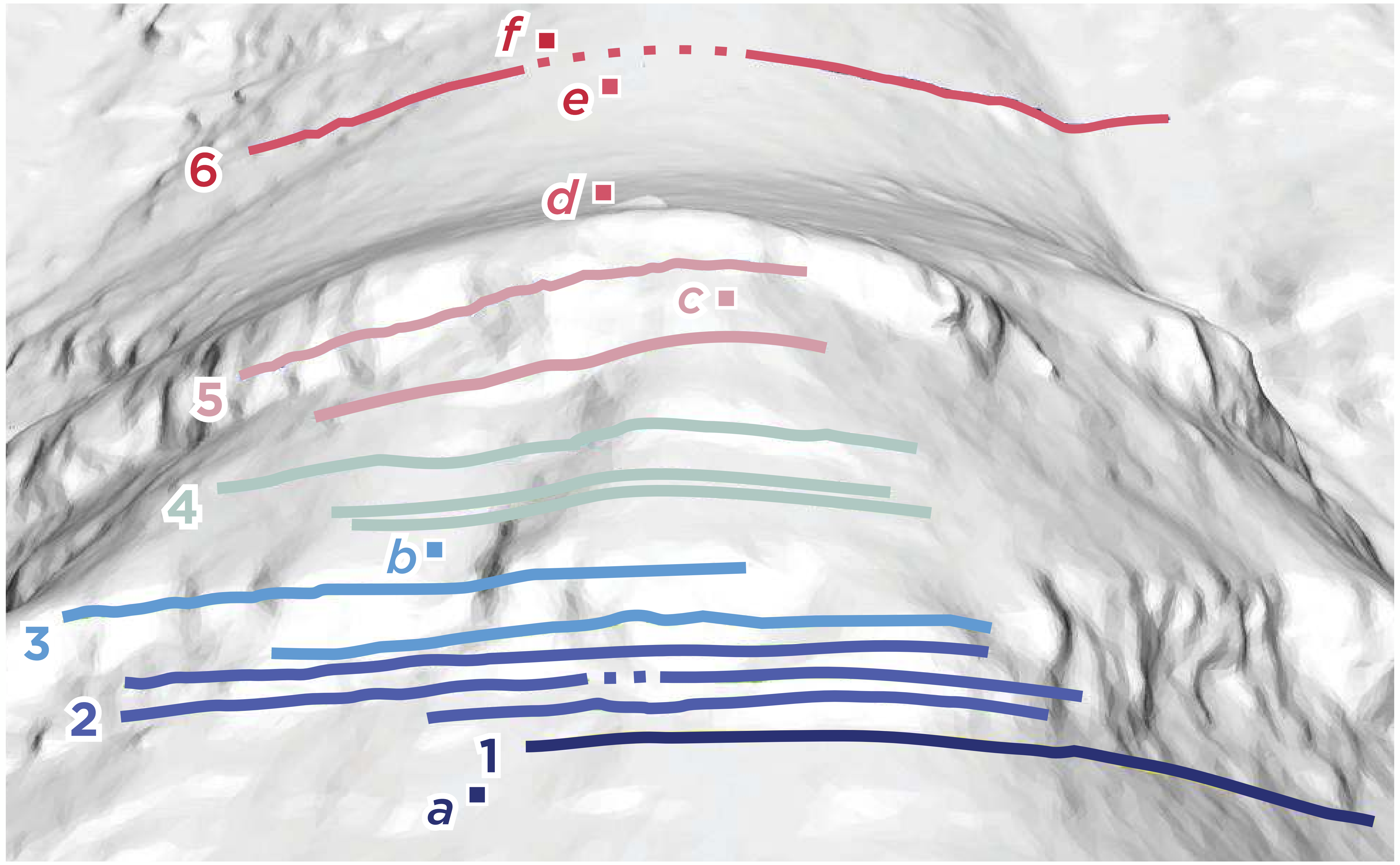


Results

- Good correspondence between directly-measured and remotely-sensed orientations
- Subtle folding of stratigraphy is confirmed by both UAV and direct measurements
- High-quality measurements without leaving Photoscan (within typical workflow for UAV data)



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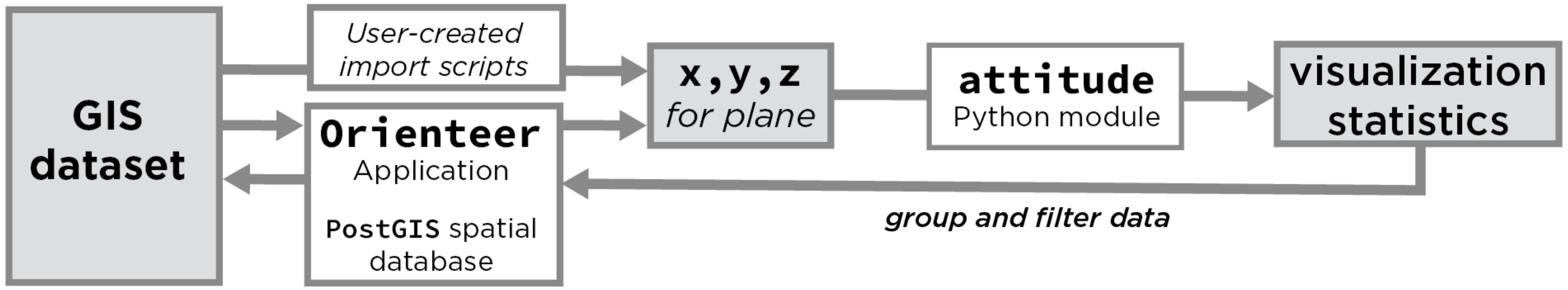
Supporting software



1. `attitude`

https://github.com/davenquinn/attitude

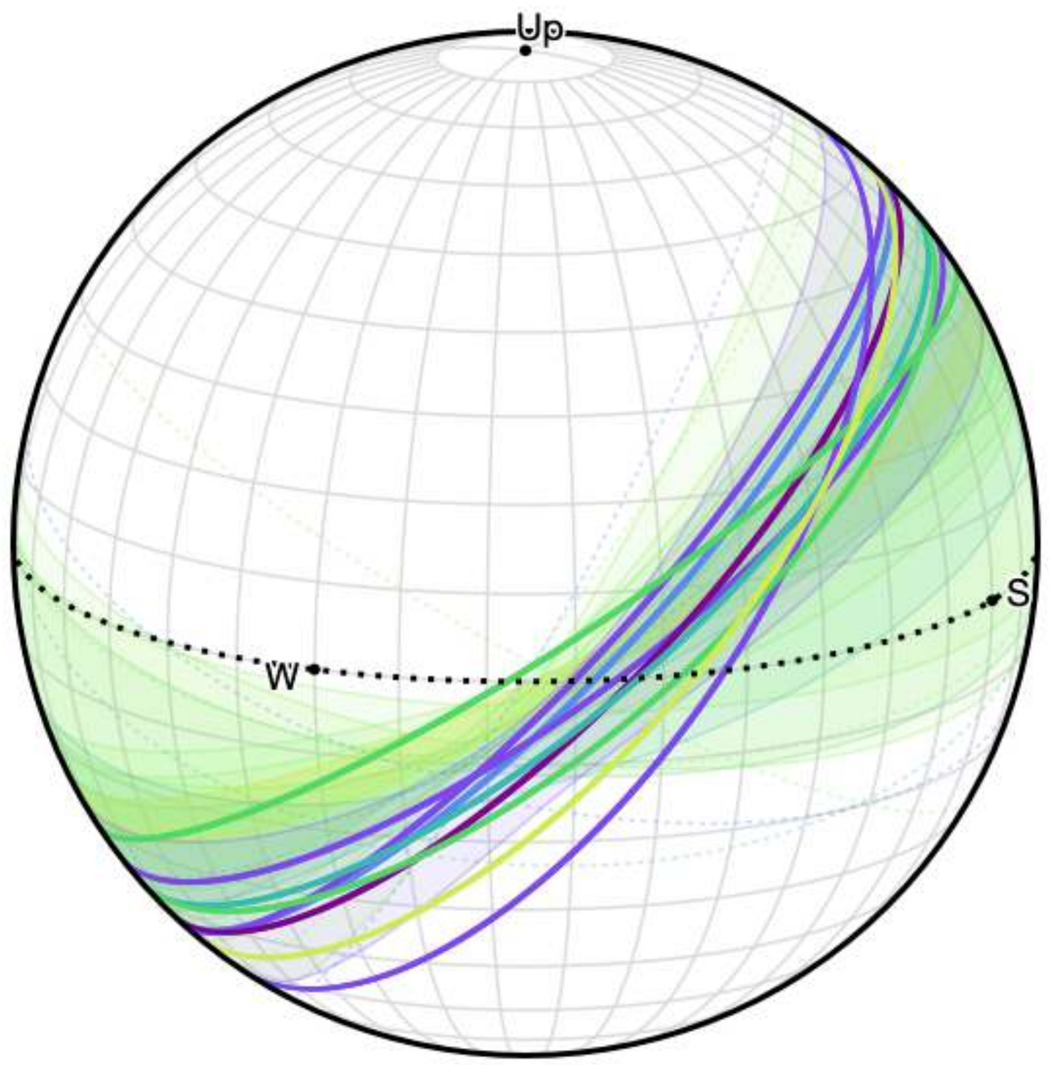
https://davenquinn.github.io/attitude (documentation)

- Python module implementing math and helper functions
- Javascript components for displaying data on stereonets
- Compatibility with iPython notebooks
- Core of the method
- Simple to install and use

```
heights = N.array([o['center'][2] for o in col
rng = [heights.min(),heights.max()]

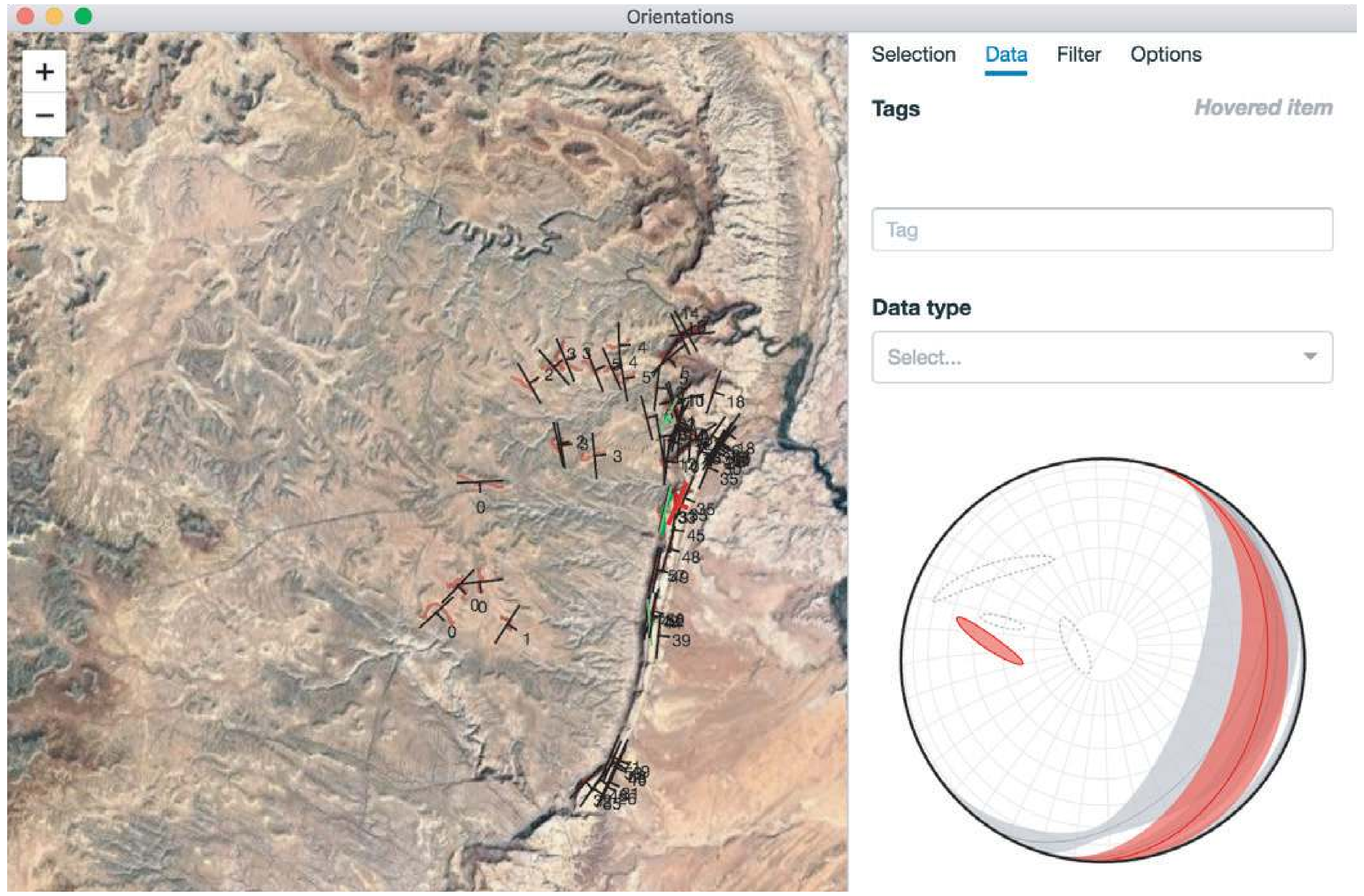
for o in collection:
    ix = N.interp(o['center'][2], rng, [0,6])
    o['color'] = cmap.hex_colors[6-int(ix)]
```

```
init_notebook_mode()
plot_interactive(collection)
```



2. `Orienteer`

- Data-management application for regional mapping
- Stores orientations in a PostGIS database and supports interactive grouping and data exploration
- Steeper learning curve than `attitude`



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Conclusions

https://github.com/davenquinn/attitude

- A new statistical framework for error analysis of geologic plane orientations
  - Based on principal component analysis (PCA)
  - Responsive to different sources of error and view geometries
  - Supports joint fitting of parallel sedimentary bedding
- Tested and adaptable to terrestrial and planetary data at a variety of scales
- Particularly well-suited to structure-from-motion photogrammetry
- Software tools for the visualization of orientation errors

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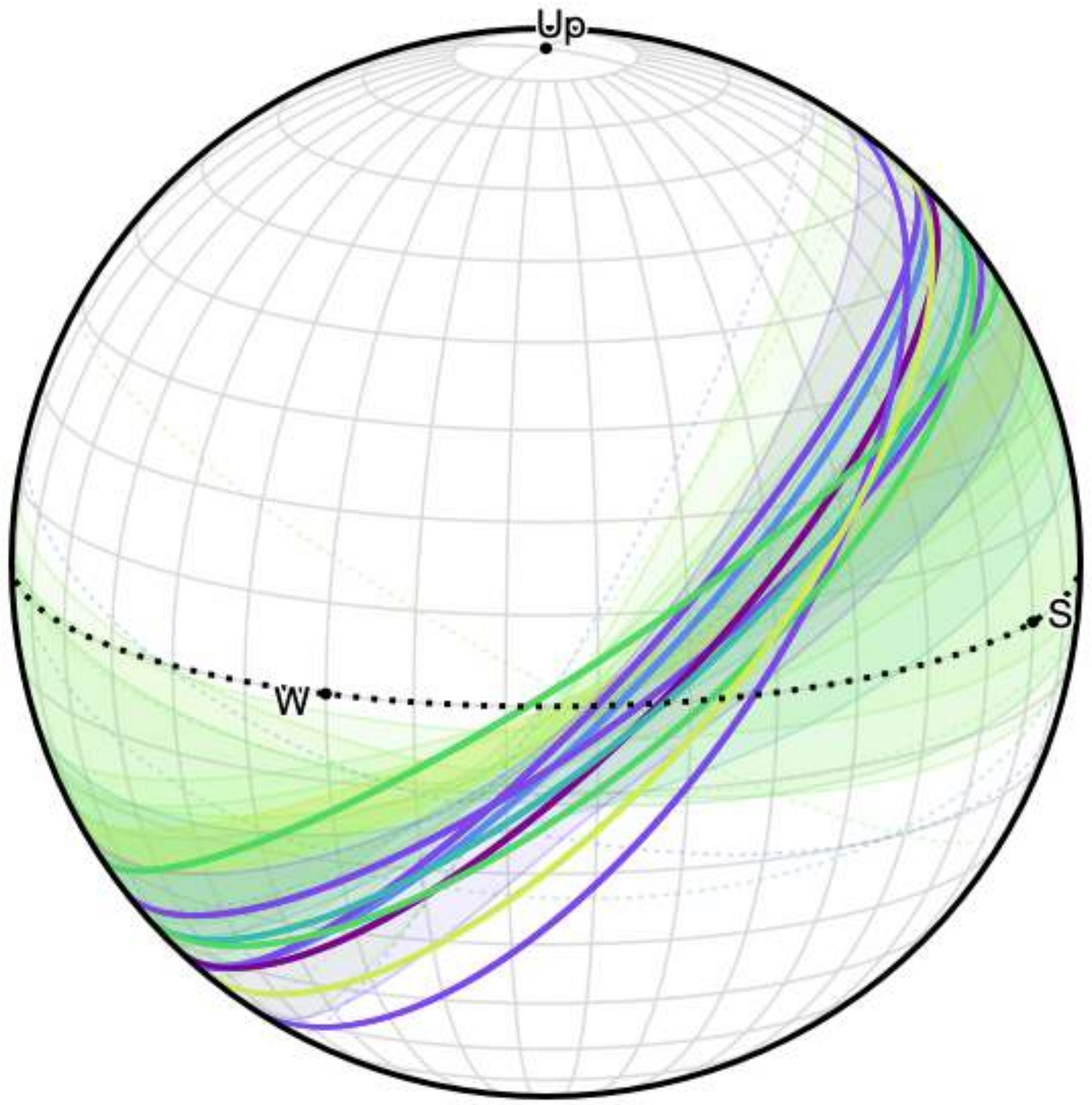
```

Next steps

- Use it!
- Help improve the software, methods, and documentation
- Re-implement the statistical transform in a language of your choice



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