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Evaluation of Transfer Function Noise Modelling and Dimensionality Reduction Techniques for Karst Systems

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Motivation and Aim

Transfer Function Noise (TFN) Modelling

- **Modelling** and forecasting karst system spring discharge **still poses a challenge**
- **Distributed modelling** approaches often suffer from **insufficiently available data**
- **Lumped parameter** models often do not reflect **physical system understanding**

→ Development of a **conceptually and (partially) physically interpretable** data-driven model

Dimensionality Reduction (DR)

- Difficulties in **physical model interpretability** pose challenges regarding **model calibration and verification** of parameters
- **Active subspaces** (AS, linear DR) offer the possibility of studying the **model input-output map** from the parameter space to a scalar model output, revealing important directions in the parameter space to reduce the dimensionality

→ Investigation of **parameter and process relationships** in the AS framework

The **suitability of the combined methods of TFN modelling and DR** is systematically evaluated w.r.t. the **capability of simulating karst system spring discharge** and the assessment of **parameter and process relationships**.

Materials and Methods

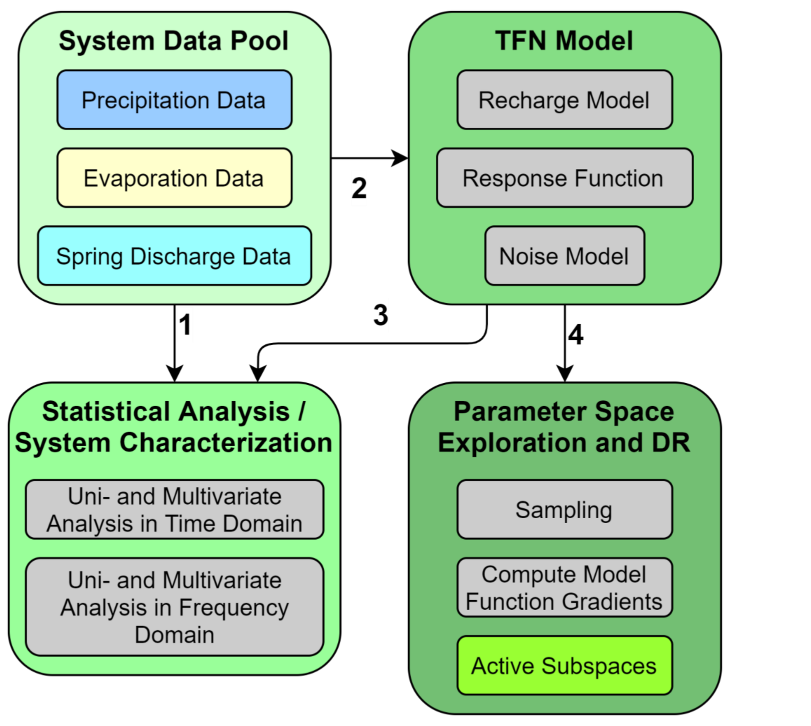

Fig. 1: Complete Workflow Applied for Synthetic Sys. & Milandre Karst System

Considered Systems

- Milandre Karst System, heavily studied, subject to *Karst Modelling Challenge* (KMC) → comparison of TFN model with other approaches
- Three synthetic systems reacting on different time scales (MODFLOW + CFP + recharge model (RM, Fig. 2))

Setup of the Combined TFN modelling and DR approach

- TFN model tested with different response function and recharge model combinations
- Residuals are modelled with an AR(1) noise model → more robust parameter inference
- Model calibration with a least-squares solver → residuals / noise minimized
- Parameter space exploration with *Markov chain Monte Carlo* (MCMC), computation of model output (linearized NSE-criterion) gradients w.r.t. parameters

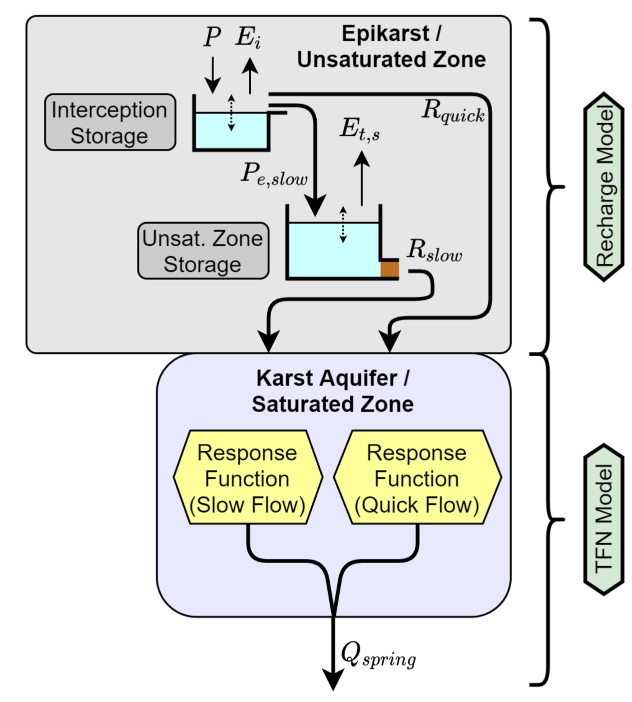

Fig. 2: Non-Linear Recharge Model – Adapted After *Collenteur et al. (2021)*

Results – Milandre Karst System

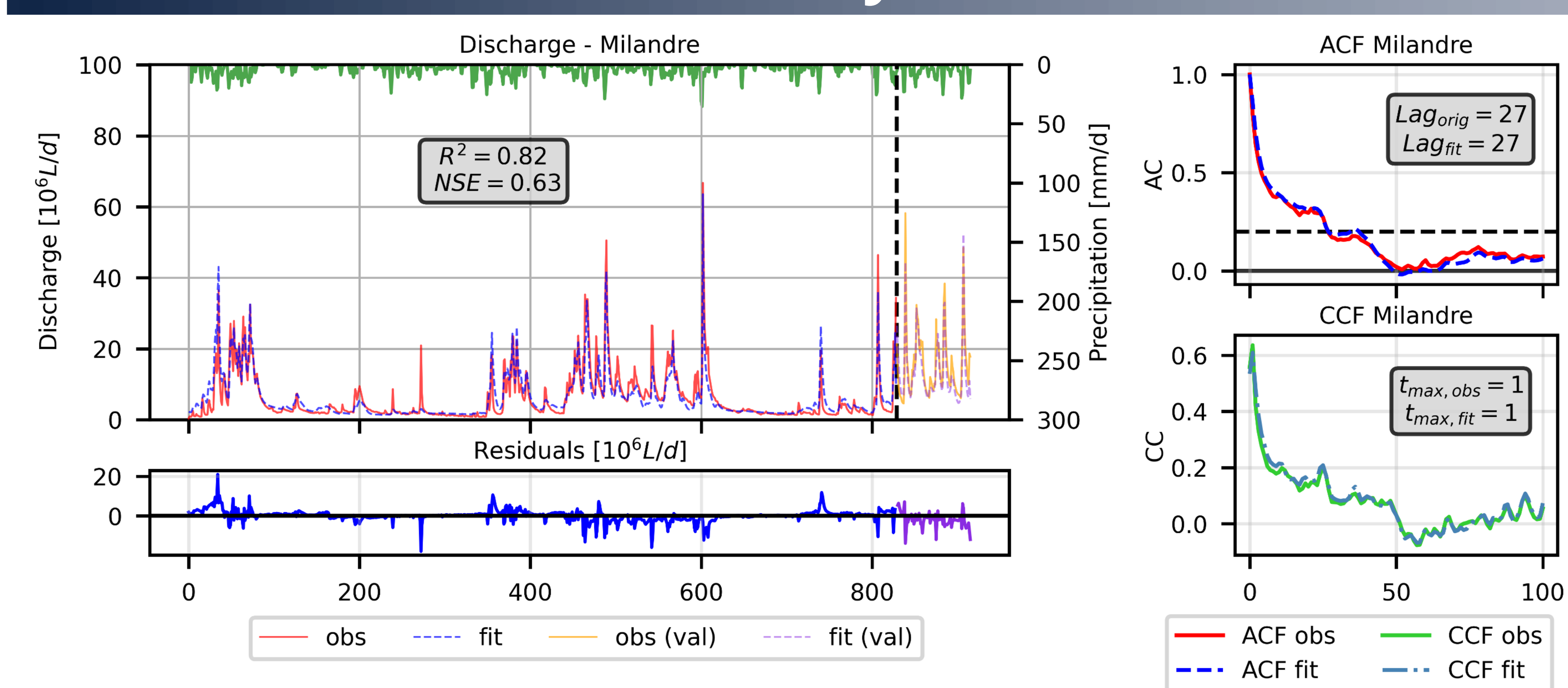

Fig. 3: Observed and Simulated Spring Discharge, Autocorrelation (ACF) and Cross-Correlation (CCF) Functions for the Milandre Karst System

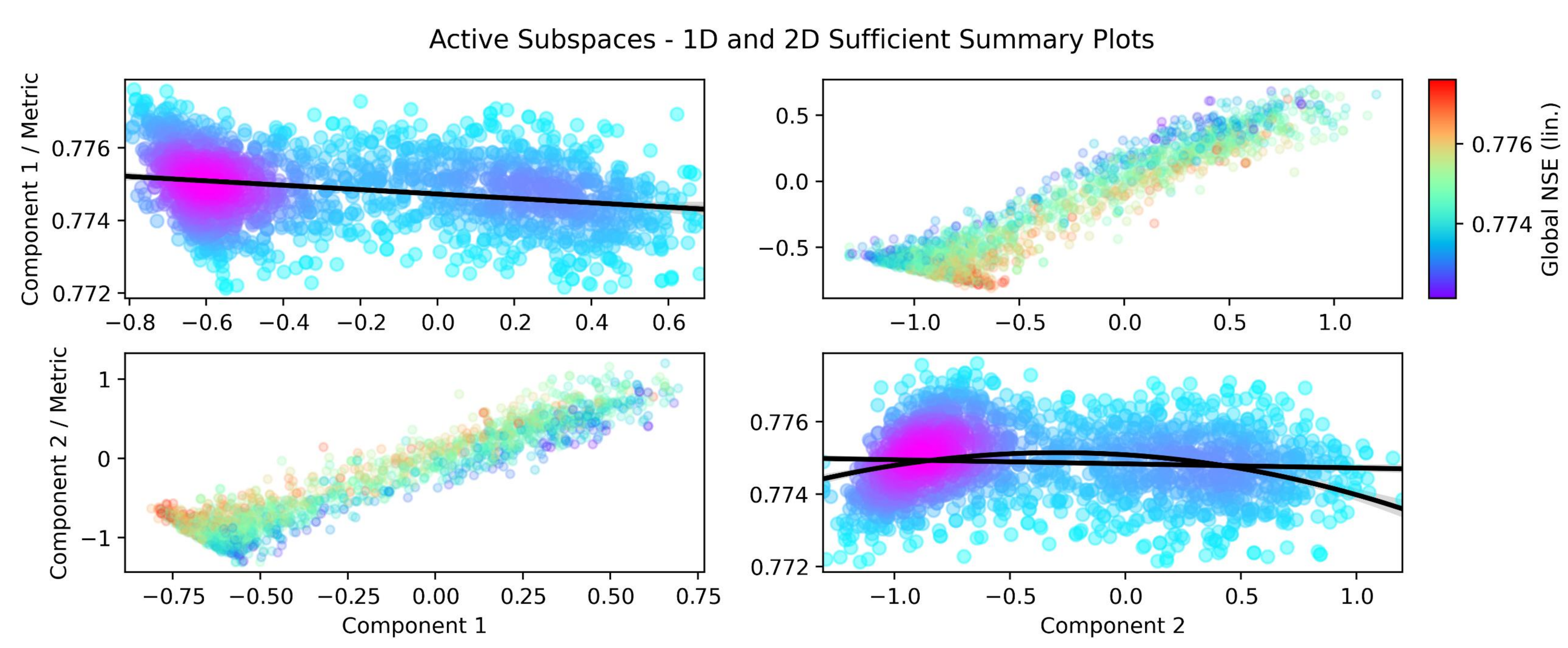

Fig. 4: 1D and 2D Sufficient Summary Plots Representing the Parameter Samples in the Active Subspace and Relationships with the Model Output; Colors on the Main Diagonal Represent Point Density

TFN Modelling

- Non-linear RM needed for satisfactory fit and preservation of system characteristics (Fig. 3)
- RM compensates for system non-linearity
- modelled processes may not be physically measurable
- Approach outperformed 11 out of 13 other models in the KMC (see *Jeannin et al. (2021)*)

Dimensionality Reduction

- Sensitivity (Fig. 5) of diffuse response shape parameters reflects real system functioning (mainly diffuse recharge for Milandre)
- 2 active dimensions / linear combinations identified (from 13 original parameters)
- Clear two-dimensional relationship between parameter space and model output (Fig. 4)

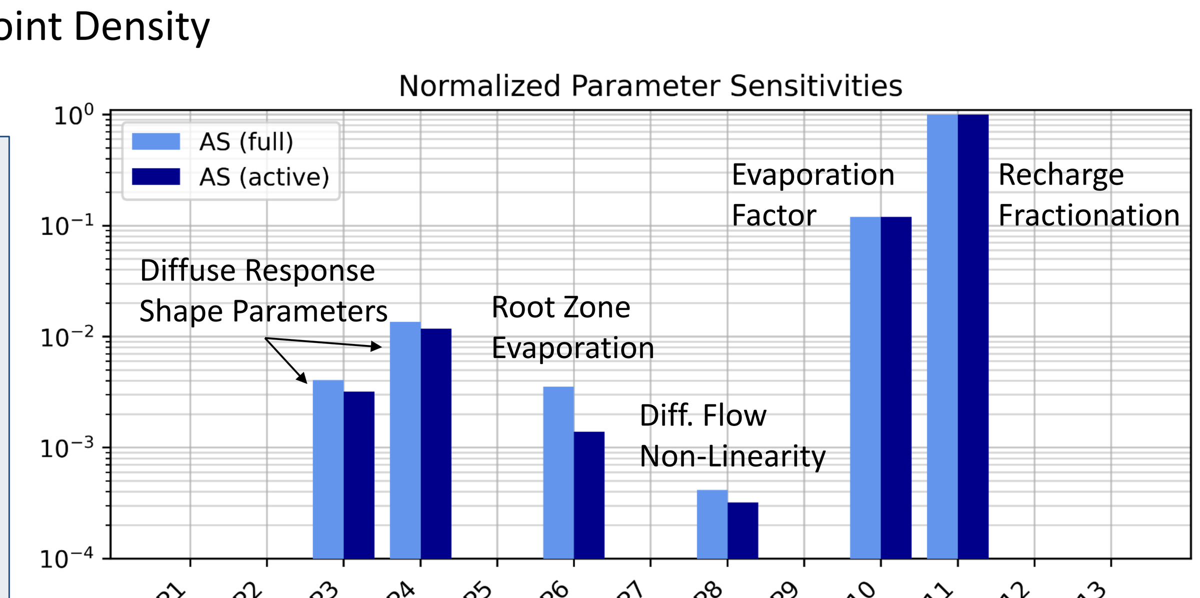

Fig. 5: Normalized Parameter Sensitivities in the Full Input Space and in the Active Subspace

Summary and Outlook

TFN Modelling

- System non-linearity gets represented as part of recharge process → difficult physical interpretability
- TFN model performed very good compared to other approaches of the KMC
- Approach is suitable for representation of karst system spring discharge

Dimensionality Reduction

- Process and parameter relationships could be revealed to characterize the modelling framework and the studied system
- Lower dimensional structures always identified → beneficial for subsequent use (surrogate models, parameter inference in low-dimensional setting etc.)

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