Evaluation of Transfer Function Noise Modelling and Dimensionality Reduction Techniques for Karst Systems

Motivation and Aim

Transfer Function Noise (TFN) Modelling
- Modelling and forecasting karst system spring discharge still poses a challenge
- Distributed approaches often suffer from insufficiently available data
- Lumped parameter models often do not reflect physical system understanding
- Development of a conceptually and (partially) physically interpretable data-driven model

Dimensionality Reduction (DR)
- Difficulties in physical model interpretability pose challenges regarding model calibration and verification of parameters
- Active subspaces (AS, linear DR) offer the possibility of studying the model input-output map from the parameter space to a scalar model output, revealing important directions in the parameter space to reduce the dimensionality
- Investigation of parameter and process relationships in the AS framework

The suitability of the combined methods of TFN modelling and DR is systematically evaluated w.r.t. the capability of simulating karst system spring discharge and the assessment of parameter and process relationships.

Materials and Methods

Considered Systems
- Milandre Karst System, heavily studied, subject to Karst Modelling Challenge (KMC) compared to TFN model with other approaches
- Three synthetic systems reacting on different time scales (MODFLOW + CFP + recharge model, RM, Fig. 2)

Setup of the Combined TFN modelling and DR approach
- TFN model tested with different response function and recharge model combinations
- Residuals modelled with an AR(1) noise model more robust parameter inference
- Model calibration with a least-squares solver to residuals/noise minimized
- Parameter space exploration with Markov chain Monte Carlo (MCMC), computation of model output (linearized NSE-criterion) gradients w.r.t. parameters

Results – Milandre Karst System

TFN Modelling
- Non-linear RM needed for satisfactory fit and preservation of system characteristics (Fig. 3)
- RM compensates for system non-linearity
- Modelled processes may not be physically measurable
- Approach outperformed 11 out of 13 other models in the KMC (see Jeannin et al. (2021))

Dimensionality Reduction
- Sensitivity (Fig. 5) of diffuse response shape parameters reflects real system functioning (mainly diffuse recharge for Milandre)
- 2 active dimensions/linear combinations identified (from 13 original parameters)
- Clear two-dimensional relationship between parameter space and model output (Fig. 4)

Summary and Outlook

TFN Modelling
- System non-linearity gets represented as part of recharge process difficult physical interpretability
- TFN model performed very good compared to other approaches of the KMC
- Approach is suitable for representation of karst system spring discharge

Dimensionality Reduction
- Process and parameter relationships could be revealed to characterize the modelling framework and the studied system
- Lower dimensional structures always identified beneficial for subsequent use (surrogate models, parameter inference in low-dimensional setting etc.)

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